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Pietro Senesi

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Abstract

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*Department of Human and Social Sciences, University of Naples "L'Orientale", Largo S. Giovanni Maggiore, 30 - 80134 Naples, Italy; email: psenesi@unior.it, Phone: +39 081 6909482, Fax: +39 081 6909442

1 Introduction

Ryan (1974) showed that "...only under highly improbable conditions concerning the probability density function of returns do unbounded utility functions fail to discriminate correctly between alternative actions." He showed that it suffices to have finite higher moments for polinomial utility functions to be admissible under all probability density functions. Also, he provided an example of a strictly concave admissible utility function. Russell and Seo (1978) provided necessary and sufficient conditions that must hold pointwise over the set of returns for a utility function to be admissible and proved that the logarithmic utility function is inadmissible even with random variables that have finite moments.

The widespread diffusion of lottery play motivates the study of utility functions defined over non zero measure sets of goods. Burger et al. (2020) provide experimental evidence that participating in a game may by itself provide utility to the players, even when no subsequent win occurs as when e.g. wins consist of extended playtime (as provided by many arcade games and lottery tickets) or flows of goods. In such instances agents can be assumed to enjoy utility on strictly positive measures of returns.

We extend the result in Russell and Seo (1978) by providing corresponding conditions when, in such lotteries the class of utility functions compatible with expected utility maximization can be expanded and the inequalities by Russell and Seo (1978) are required to hold only in measure and not pointwise.

2 The result

Take an element k of the set Ω of all Lebesgue measurable functions defined on the interval I, with m denoting Legesgue measure. Given this k, denote by χ_k the set of all random variables X such that $E[k(X)] < \infty$. ($E[k(X)] < \infty \Rightarrow X \in \chi_k$, i.e. k acts as a filter on all the Xs to create the set χ_k.) Then, given a k, $E(u(X)) < \infty \; \forall X \in \chi_k \Leftrightarrow \int_{I_n} u(x) < \infty \; \forall I_n \subset I$.

The theorem below states that the utility function need be bounded in measure by a monotone transform of the absolute value of the given moment. Thus, the utility function need not be bounded pointwise, as long as it assumes infinite values only on zero measure subsets of I. For such probability densities, even the logarithmic utility function is admissible.

Theorem 1. *Let u be a Lebesgue measurable function defined on I. Then for each $k \in \Omega$, $E[u(X)] < \infty \; \forall X \in \chi_k$, if and only if there exists some positive real numbers M and N such that*

$$\int_{I_n} |u(x_n)| \leq M \cdot \int_{I_n} |k(x_n)| + N, \; \forall I_n \in I, \; m(I_n) > 0. \tag{1}$$

Proof. Sufficiency. Equation (1) implies

$$\int_I |u(x)| \leq M \cdot \int_I |k(x_n)| + N. \tag{2}$$

Necessity. In order to obtain a contradiction, assume there exists a sequence of intervals $I_n \in I$ such that

$$\int_{I_n} |u(x_n)| \geq n^2 \left(1 + \int_{I_n} |k(x_n)|\right) \tag{3}$$

$$\int_{I_n} |u(x_n)| \geq M \cdot \int_{I_n} |k(x)| + N \tag{4}$$

$\forall I_n \in I, x \in I_n$.

$$g(x_n) = \frac{1}{n^2 \left(1 + \int_{I_n} |k(x_n)|\right)} \tag{5}$$

$$A = \sum_{n=1}^{\infty} g(x_n) \tag{6}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^2 \left(1 + \int_{I_n} |k(x_n)|\right)} \tag{7}$$

$$\tag{8}$$

$$p(x) = \frac{g(x_n)}{\sum_{n=1}^{\infty} g(x_n)} \tag{9}$$

for $x = x_n, n = 1, 2, \ldots,$ and $p(x) = 0$ otherwise.
Then

$$F(x) = \sum_{x_n < n} p(x_n), x \in (-\infty, \infty) \tag{10}$$

is a c.d.f. Then

$$\int_{-\infty}^{\infty} k(x) dF = E[|k(x)|] = \sum_{n=1}^{\infty} [|k(x_n)| \cdot p(x)] \tag{11}$$

$$= \sum_{n=1}^{\infty} \left[\int_{I_n} (|k(x_n)|) \cdot \frac{1}{An^2(1 + \int_{I_n} |k(x_n)|)} \right] \tag{12}$$

Since

$$\theta = \left(\frac{\int_{I_n} |k(x_n)|}{1 + \int_{I_n} |k(x_n)|} \right) < 1 \tag{13}$$

it holds that

$$\sum_{n=1}^{\infty} \frac{1}{An^2} \cdot \theta < \sum_{n=1}^{\infty} \frac{1}{An^2} = A^{-1} \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty \tag{14}$$

so that the random variable X with c.d.f. F has finite moment $E[k(x)]$, i.e. it belongs to χ_k.

But from (1)

$$E[|u(x)|] = \sum_{n=1}^{\infty} \left[\frac{\int_{I_n} |u(x_n)|}{An^2(1 + \int_{I_n} |k(x_n)|)} \right] \geq \sum_{n=1}^{\infty} \left[\frac{n^2(1 + \int_{I_n} |k(x_n)|)}{An^2(1 + \int_{I_n} |k(x_n)|)} \right] \tag{15}$$

$$= \sum_{n=1}^{\infty} A^{-1} = \infty \tag{16}$$

or

$$E[|u(x)|] = \sum_{n=1}^{\infty} \left[\frac{\int_{I_n} |u(x_n)|}{An^2(1 + \int_{I_n} |k(x_n)|)} \right] \geq \sum_{n=1}^{\infty} \left[\frac{n^2(1 + \int_{I_n} |k(x_n)|)}{An^2(1 + \int_{I_n} |k(x_n)|)} \right] \tag{17}$$

$$= \sum_{n=1}^{\infty} A^{-1} = \infty \tag{18}$$

□

In the above proof, the term $|k(x_n)|$ can be substituted for $\int_{I_n} |k(x_n)|$, as the integral of a bounded function over a set of bounded measure is bounded.

References

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