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Abstract

We study the effects of factor income taxation on the dynamics of equilibrium paths. When net of taxes returns are treated as fiscal parameters, endogenous fluctuations may emerge as a consequence of Hopf bifurcations. A technique based on the rank of Bezout matrices for polynomials associated to the Jacobian matrix is used to compute the Hopf points.

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1 Introduction

The emergence of cyclical dynamics is a result displayed by several models of dynamic economies. In the multisector model of optimal economic growth, the occurrence of a Hopf bifurcation is a well established result (see Benhabib and Nishimura [2], Medio [14], Benhabib and Rustichini [3] and Cartigny and Venditti [6]). The result hinges on the evolution of relative prices being sufficient to make it optimal to invest and disinvest in a cyclical manner. Following Iooss and Joseph [9], Zhang [18] provides analytical tools for the construction of the periodic solutions as power series amplitude equations. The present paper addresses computational aspects of Hopf points in the

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standard model of second-best efficient taxation of factor income put forth by Chamley [7]. We investigate the relationship between the use of factor taxes as fiscal parameters in the steady state and the possibility of induced endogenous fluctuations, especially the computational aspect. Under this respect, we depart from the standard literature on the optimal policy mix (see, e.g. Atkinson and Sandmo [1], Chamley [7], Jones et al. [10], Judd [11], [12], Mirrlees [15], Ordover and Phelps [16], and Phelps [17]) where the tax rates are always designed so as to solve the intertemporal maximization problem.

Our interest does not lie in the construction of amplitude equations nor in investigating the legitimacy of simulations when the dynamical system is not hyperbolic. Rather we focus on techniques for the computation of Hopf points, i.e. where the necessary conditions for the Hopf bifurcation are met.

The paper is organized as follows: Section 1 reviews the standard model of factor income taxation. Section 2 characterizes conditions for the existence of Hopf points. Section 3 deals with the computation of the Hopf points by using a technique based on the rank of Bezout matrices for polynomials associated to the Jacobian matrix. Section 4 ends the paper with a summary of the results.

2 The model

The following analysis is framed inside the standard model of second-best taxation of Chamley [7]. Instantaneous utility depends on both consumption and labor effort,

$$u(c,l) \triangleq \frac{c^{1-\sigma}}{1-\sigma} + L(1-l) \tag{1}$$

where σ is a strictly positive parameter, and L is utility from leisure. The functions L and u are such that the current value Hamiltonian of the second-best problem can be written as

$$\begin{aligned} H &= \upsilon(\bar{w},q) + \xi q(\rho - \bar{r}) + \lambda\left[f(k,l) - c - g\right] \\ &\quad + \mu\left[\bar{r}b + \bar{r}k + \bar{w}l - f(k,l) + g\right] + \nu\bar{r} \end{aligned} \tag{2}$$

where υ is an indirect utility function; the private subjective time preference rate is ρ; $\bar{r}$ and $\bar{w}$ denote net-of-taxes interest and wage rates, respectively;

the standard constant returns to scale production function is f; b is government debt, k is capital, g is government spending, ξ, λ, μ, and ν are Lagrange multipliers.

Net investment is

$$\dot{k} = f(k, l) - c - g \tag{3}$$

and

$$\dot{b} = \bar{r}b + \bar{r}k + \bar{w}l - f(k, l) + g \tag{4}$$

is the instantaneous increase in public debt.

The constraints

$$u_c^{'} = q, \ \ u_l^{'} = -q\bar{w} \tag{5}$$

and

$$\dot{q} = (\rho - \bar{r})\, q \tag{6}$$

ensure that the planner's solution meets the conditions for static and dynamic utility maximization on the part of private agent. Lump-sum taxation of capital income is ruled out by the constraint

$$\bar{r} \geq 0 \tag{7}$$

binding for $t \in [t_0, \tau_1]$.

The efficient policy-mix satisfies the three equations

$$\dot{\xi} = \rho\xi - \frac{\partial H}{\partial q}$$

$$\dot{\xi} = \rho\xi - v_q' - \lambda\left(l_q' f_l' - c_q'\right) - \mu l_q' \left(\bar{w} - f_l'\right) \tag{8}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial k}$$

$$\dot{\lambda} = \lambda\left(\rho - f_k'\right) - \mu\left(\bar{r} - f_k'\right) - \frac{\partial H}{\partial k} \tag{9}$$

and

$$\dot{\mu} = (\rho - \bar{r})\, \mu \tag{10}$$

$$\frac{\partial H}{\partial \bar{w}} = 0$$

$$\frac{\partial H}{\partial \bar{r}} = -\xi q + \mu(b + k) + \nu = 0$$

3 Hopf points

The Jacobian matrix in the vicinity of the steady state is (see Chamley [7])

$$J = \begin{bmatrix} r + wl_k & wl_q - c_q & 0 \\ -qr_k - qr_l l_k & -qr_l l_q & 0 \\ \cdot & \cdot & \rho \end{bmatrix} \tag{11}$$

The following theorem states conditions under which such a growth path experiences a Hopf bifurcation into periodic orbits during the initial phase of efficient strictly positive capital income taxation. When this is the case, the dynamics display endogenous perpetual fluctuations and do not converge to the steady state. Moreover, the dynamics are characterized by a strictly positive capital income tax and are conservative, thus repeating themselves over and over to infinity with the same periodicity (unless exogenous shocks move the economy to a different orbit). In this state, the tax on capital income keeps efficient always, in the long-run as well. Chamley [7] result is restricted to the class of second-best efficient economies which have a stationary long-run dynamics. The following characterizes a class of economies with endogenous perpetual fluctuations where the long-run tax rate on capital income is not zero.

Theorem 1 *Assume*

a) $-q(r_k + r_l l_k)(wl_q - c_q) < (r + wl_k)(-qr_l l_q)$ *in a neighborhood of* $r = -wl_k + qr_l l_q$;

b) $\left[\frac{\partial \xi(r)}{\partial r}\right]_{r=r^*} \neq 0$.

Then the system $\left(\dot{k}, \dot{q}\right)$ *undergoes a Hopf bifurcation.*

Proof. The sub-matrix

$$M = \begin{bmatrix} r + wl_k & wl_q - c_q \\ -qr_k - qr_l l_k & -qr_l l_q \end{bmatrix} \tag{12}$$

has eigenvalues which are solutions to the polynomial equation

$$\varsigma^2 - (a + d)\varsigma + ad - bc = 0 \tag{13}$$

where

$$
\begin{aligned}
a &\triangleq r + wl_k && (14)\\
d &\triangleq -qr_l l_q && (15)\\
b &\triangleq wl_q - c_q && (16)\\
c &\triangleq -qr_k - qr_l l_k && (17)
\end{aligned}
$$

So we may write

$$\varsigma_{1,2} \triangleq \xi(\delta) \pm i\eta(\delta) \tag{18}$$

where

$$\xi(\delta) \triangleq a + d = r + wl_k - qr_l l_q \tag{19}$$

At the Hopf point, $r = r^*$ such that

$$\xi(r^*) = 0, \tag{20}$$

This implies

$$d = -a \Leftrightarrow r = -wl_k + qr_l l_q \tag{21}$$

Then (13) reduces to

$$\varsigma^2 + ad - bc = 0 \tag{22}$$

and

$$\varsigma_{1,2} = \pm i\eta(\delta) \triangleq \pm(bc - ad)^{1/2} \tag{23}$$

where

$$bc - ad = -q(r_k + r_l l_k)(wl_q - c_q) + (r + wl_k)(qr_l l_q) \tag{24}$$

For the eigenvalues to be purely imaginary, it is needed

$$-q(r_k + r_l l_k)(wl_q - c_q) < (r + wl_k)(-qr_l l_q) \tag{25}$$

Moreover

$$\left[\frac{\partial \xi(r)}{\partial r}\right]_{r=r^*} \neq 0 \tag{26}$$

(see Iooss and Joseph [9]). ■

The weakness of this theorem is that it relies on assumptions which are not easy to give a clear economic meaning to and that capital income taxation seems to be zero on average.

4 Computation of Hopf points

The Jacobian matrix in the vicinity of the steady state is (see Chamley [7])

$$J=\begin{bmatrix} r+wl_k & wl_q-c_q & 0 \\ -qr_k-qr_l l_k & -qr_l l_q & 0 \\ \cdot & \cdot & \rho \end{bmatrix} \tag{27}$$

One eigenvalue is ρ and is always positive. The dot represents a nonzero element. Hence we confine our study to the submatrix (12)

$$\begin{bmatrix} r+wl_k & wl_q-c_q \\ -qr_k-qr_l l_k & -qr_l l_q \end{bmatrix}$$

The coefficients of the matrix depend on the fiscal parameters. Its characteristic polynomial is

$$p(\varsigma)=c_0+c_1\varsigma+c_2\varsigma^2 \tag{28}$$

The nonzero vector $(\varsigma,-\varsigma)$ is a root pair of (28) if and only if ς is a common root of the equations $p(\varsigma)+p(-\varsigma)$ and $p(\varsigma)-p(-\varsigma)$. Letting $z=\varsigma^2$ we have the system

$$\begin{aligned} r_e(z) &= c_0+c_2 z \\ r_0(z) &= c_1+c_3 z \end{aligned} \tag{29}$$

Clearly, $p(\varsigma)$ has a nonzero root pair $(\varsigma,-\varsigma)$ if there exists z such that

$$\begin{pmatrix} r_e(z) \\ r_0(z) \end{pmatrix}=0$$

Let B denote the Bezout matrix associated with $(r_e(z), r_0(z))$. Then, straightforward application of Theorem 2.5 in Guckenheimer et al. [8] gives

Theorem 2 *The matrix J has precisely one pair of pure imaginary eigenvalues if $det(B)=0$ and $det(B_0)\cdot det(B_1)>0$. If $det(B)\neq 0$ or $det(B_0)\cdot det(B_1)<0$, then $p(\lambda)$ has no purely imaginary roots.*

Proof. (the proof is given here for ease of reference) ■

5 Conclusions

We study the effects of factor income taxation on the dynamics of equilibrium paths. When net of taxes returns are treated as fiscal parameters, endogenous fluctuations may emerge as a consequence of Hopf bifurcations. A technique based on the rank of Bezout matrices for polynomials associated to the Jacobian matrix is used to compute the Hopf points.

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