The efficiency of the Friedman rule in a Monetary Union

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Abstract

This paper evaluates the performance of the Friedman rule and characterizes efficient allocation of resources within a new-generation, search-theoretic model of monetary union with divisibility and degenerate distribution of money holdings. The main results are: *i)* the Friedman rule is a necessary and sufficient condition for global efficiency, and *ii)* the Friedman rule is necessary for local efficiency, as it is the unique policy rule that eliminates the hold-up problem in any local market. It is also shown that the efficient quantity demanded by a country-i buyer is increasing, and the country-i price of goods is decreasing, in the country-i fraction of sellers. (*JEL Classification: D83, E40, E50*).

Keywords: money, search, Friedman rule, monetary union

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1 Introduction

This paper evaluates the performance of the Friedman rule and characterizes efficient allocation of resources within a new-generation, search-theoretic model of monetary union with divisibility and degenerate distribution of money holdings.

Previous works on this topic have shown that a rate of growth of money that obeys the Friedman rule may or may not guarantee efficiency.[1] The Friedman rule generates the first-best allocation in the Shi's [17] households model, whereas it only achieves the second-best in Lagos and Wright [13] (hereafter, LW), unless eithrt buyers have all the bargaining power or agents are price takers in the decentralized trade market (see [15], [16]).[2]

In their analysis, LW assume a unique decentralized market, which in turn implies a unique strategy across agent-types (i.e. buyers, sellers). This paper extends their setup by assuming that two disjoint groups of agents participate in two distinct decentralized local markets before they can rebalance their money holdings in the centralized market; it turns out that agents of the same type may have different strategies if trading conditions differ across local markets. This in turn permits to study the behavior of prices and efficiency in a two-country monetary union.

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The remainder of the paper is organized as follows. Sections 2 and 3 present and extend the LW model. Equilibria are characterized in Section 4. Section 5 describes results. Section 5 ends the paper with a brief summary.

[1] Inefficiencies associated with search frictions have been analyzed in literature by [1], [5], [7], [13], [14], [15], and [16].

[2] The Shi and LW frameworks use different framework to obtain a degenerate distribution of money. For a detailed discussion of the two approaches see [12].

2 The model

The framework of analysis is taken from LW. Time is discrete and tends to infinity. There is a $[0,1]$ continuum of agents and one perishable good that can be produced and consumed. The economy consists of two countries labelled $i \in \{1,2\}$. Let α_i be the fraction of total population living in country i. Each period is divided into two subperiods, morning and afternoon. In the morning, agents living in country i trade bilaterally in a decentralized market, called local market i, while in the afternoon trade occurs in a centralized and frictionless market open to all agents regardless of their nationality; this market is called international market.

When the local market starts, agents are subject to a preference shock such that they can either consume or produce as follows. With probability $1-n_i$ an agent of country i can consume but cannot produce, while with probability n_i the agent can produce but cannot consume.[3] We refer to consumers as buyers, and to producers as sellers. Agents get utility $u(q)$ from q units of consumption in the local market, with $u'(q)>0$, $u''(q)<0$, $u'(0)=\infty$, and $u'(\infty)=0$. Furthermore, we assume that the elasticity of utility $e(q)=qu'(q)/u(q)$ is bounded. Producers incur a utility cost $c(q)$ from producing q units of output, with $c'(q)>0$, $c''(q)>0$, $c'(0)=0$ and $c'(\infty)=\infty$. Let q^* denote the solution to $u'(q^*)=c'(q^*)$. To make money essential, it is assumed that buyers in the local market are *anonymous*. Consequently, trading histories of agents are private information and trades are subject to a quid pro quo restriction ([9], [18]).

In the international market all agents consume and produce, getting utility $U(x)$ from x units of consumption, with $U'(x)>0$, $U'(0)=\infty$, $U'(\infty)=0$ and $U''(x)\leq 0$. Let x^* be the solution to $U'(x^*)=1$. All agents can produce consumption goods from labor using a linear technology. This implies that all agents of a given type will chose to carry the same amount of money out of the international market, independent of their trading history. Agents discount between afternoon and the next-period morning but not between morning and afternoon. This is not restrictive since as in [16] all that matters is the total discounting between one period and the next.

Let the quantity of money at time t be $M_t>0$ and assume $M_t=\gamma M_{t-1}$, where $\gamma>0$ is constant and new money is injected in case $\gamma>1$, or with-

[3] Some recent attempts to endogenize the fraction of agents entering in the market include [7], [10], [14], [15], and [17].

drawn if $\gamma < 1$, as lump-sum transfers $\delta M_{t-1} = (\gamma - 1)M_{t-1}$ to all buyers at the start of local market. To guarantee existence of a unique steady-state monetary equilibrium, γ is restricted to be larger than the discount factor $\beta \in (0, 1)$. Let $\delta_b M_{t-1} = \delta M_{t-1} / \sum_{i=1}^{2} \alpha_i (1 - n_i)$ denote the per buyer money transfer. In order to use a convenient notation, the time subscript t is omitted and $t + 1$ is shortened to $+1$, etc ...

In period t, let $\phi = 1/P$ be the real price of money and P the price of goods in the international market. Consider a stationary monetary equilibrium where aggregate real money balances are time invariant,

$$\phi M = \phi_{-1} M_{-1} \tag{1}$$

which implies that $\phi_{-1}/\phi = M/M_{-1} = \gamma$.

Let $V(m_1)$ be the expected value from trading in the local market with m_1 money balances conditional on the aggregate shock. Let $W(m_2)$ denote the expected value from entering the international market with m_2 units of money.

3 The Period

In what follows, we look at a representative period t and work backwards from the afternoon to the morning market.

3.1 Afternoon

In the afternoon agents produce h goods (equal labor supply), consume x, and adjust their money balances. The representative country-i agent's problem is

$$W_i(m_2) = \max_{x,h,m_{1,+1}} \left[U(x) - h + \beta V_i(m_{1,+1}) \right] \tag{2}$$

such that

$$x + \phi m_{1,+1} = h + \phi m_2 \tag{3}$$

where h are hours of work, $m_{1,+1}$ is the money taken into period $t + 1$, and ϕ is the price of money in terms of goods. Substituting h from (3) into (2) yields

$$W_i(m_2) = \phi m_2 + \max_{x,m_{1,+1}} \left[U(x) - x - \phi m_{1,+1} + \beta V_i(m_{1,+1}) \right]. \tag{4}$$

By strict concavity of $U(x)$ the maximizing choice of x is x^* where $U'(x^*) = 1$; the maximizing choice of $m_{1,+1}$ is obtained by the first order condition

$$\phi = \beta V'(m_{1,+1}) \tag{5}$$

where the left hand side of (5) is the marginal cost of taking money out of the international market (ϕ) and the right hand side is the marginal benefit ($\beta V'(m_{1,+1})$). Competitive markets (i.e., under price taking) and $u''(q) < 0$ are sufficient for uniqueness of $m_{1,+1}$, hence all buyers in the international market choose the same $m_{1,+1}$.[4]

Notice that the optimal choice of x is the same across time for all agents, and that $m_{1,+1}$ is independent of m_2. As a result, the distribution of money holdings is degenerate at the subsequent of the following period. This is due to the quasi-linearity assumption in (2), which eliminates the wealth effects on money demand in the international market.

The envelope condition is

$$W_{mi} = \phi. \tag{6}$$

Let q_{bi} and q_{si} denote the quantities consumed by a buyer and produced by a seller trading in local market i, respectively. Let p_i be the nominal price of goods in this market.

3.2 Morning

An agent living in country i with m_1 unit of money at the opening of local market has expected lifetime utility

$$\begin{aligned} V_i(m_1) &= (1-n_i)\left[u(q_{bi}) + W_i(m_1 + \delta_b M_{-1} - p_i q_{b_i})\right] \\ &\quad + n_i\left[-c(q_{si}) + W_i(m_1 + p_i q_{si})\right] \end{aligned} \tag{7}$$

where $p_i q_{bi}$ is the amount of money spent as a buyer, and $p_i q_{si}$ the money received as a seller.

Once the production and consumption shocks occur, agents become either a buyer or a seller. If a country-i agent is a seller in the morning, his problem is

$$\max_{q_{si}} \left[-c(q_{si}) + W_i(m_1 + p_i q_{si})\right]. \tag{8}$$

[4]See [13] and [16] for a characterization under bargaining and price posting, respectively.

The first-order condition is

$$-c'(q_{si}) + p_i W_{mi} = 0 \tag{9}$$

which, using (5), reduces to

$$c'(q_{si}) = p_i \phi. \tag{10}$$

Sellers produce an amount such that the ratio of marginal costs across markets ($c'(q_{si})$) is equal to the relative price ($p_i\phi$) of goods across markets. Due to the linearity of the envelope condition, q_{si} is independent of m_1. Consequently, sellers produce the same amount no matter how much money they hold.

If a country-i agent is a buyer in the morning, his problem is:

$$\max_{q_{bi}} \left[u(q_{bi}) + W_i(m_1 + \delta_b M_{-1} - p_i q_{bi})\right] \tag{11}$$

such that

$$p_i q_{bi} \leq m_1 + \delta_b M_{-1} \tag{12}$$

where (12) means that buyers cannot spend more money than what they bring into the local market, m_1, plus the transfer $\delta_b M_{-1}$. The buyers' first order conditions are

$$u'(q_{bi}) - p_i W_{mi} - \lambda_i p_i = 0 \tag{13}$$

and

$$\lambda_i (m_1 + \delta_b M_{-1} - p_i q_{bi}) = 0 \tag{14}$$

where λ_i is the multiplier on the cash constraint for country-i buyers. The price p_i adjusts to clear the local market i, so in equilibrium

$$(1 - n_i) q_{bi} = n_i q_{si.} \tag{15}$$

Using (6), (10), (13) and (15) the buyers first order condition reduces to

$$u'(q_{bi}) = \left(1 + \frac{\lambda_i}{\phi}\right) c'\left(\frac{1 - n_i}{n_i} q_{bi}\right) \tag{16}$$

whereas efficiency in country i is achieved at the quantity $q_{bi} = q_{bi}^*$ solving

$$u'(q_{bi}^*) = c'\left(\frac{1 - n_i}{n_i} q_{bi}^*\right). \tag{17}$$

4 Results

Differentiating (7) gets the (indirect) marginal value of money of country-i agents

$$\begin{aligned} V_i'(m_1) &= (1-n_i)\left[u'(q_{bi})\frac{dq_{bi}}{dm_1} + W_{mi}\left(1 - p_i\frac{dq_{bi}}{dm_1}\right)\right] \\ &\quad + n_i\left[-c'(q_{si})\frac{dq_{si}}{dm_1}W_{mi}\left(1 + p_i\frac{dq_{si}}{dm_1}\right)\right] \end{aligned} \tag{18}$$

which, using expressions (6) and (15) , and noting that the quantity produced by a seller does not depend on the money he holds (i.e. $\frac{dq_{si}}{dm_1} = 0$), can be rewritten as

$$V_i'(m_1) = \phi\left\{(1-n_i)\left\{\left[\frac{u'(q_{bi})}{c'\left(\frac{1-n_i}{n_i}q_{bi}\right)} - 1\right]\frac{dq_{bi}}{dm_1}p_i + 1\right\} + n_i\right\}. \tag{19}$$

Using, (1) and (5) lagged one period to eliminate $V_i'(m_1)$ from (19) one obtains

$$\frac{\gamma - \beta}{\beta} = (1-n_i)\left[\frac{u'(q_{bi})}{c'\left(\frac{1-n_i}{n_i}q_{bi}\right)} - 1\right]. \tag{20}$$

The next lemma establishes the relationship between the efficient quantity of goods demanded by country-i buyers and the country-i fraction of sellers.

Proposition 1 *The efficient quantity demanded by a country-i buyer is increasing in the country-i fraction of sellers.*

Proof. Taking the differential of (17) yields

$$u''(q_{bi}^*)\,dq_{bi}^* = c''\left(\frac{1-n_i}{n_i}q_{bi}^*\right)\frac{1-n_i}{n_i}dq_{bi}^* - c''\left(\frac{1-n_i}{n_i}q_{bi}^*\right)\frac{q_{bi}^*}{n_i^2}dn_i \tag{21}$$

or, rearranging,

$$\left[c''\left(\frac{1-n_i}{n_i}q_{bi}^*\right)\frac{1-n_i}{n_i} - u''(q_{bi}^*)\right]dq_{bi}^* = \frac{q_{bi}^*}{n_i^2}c''\left(\frac{1-n_i}{n_i}q_{bi}^*\right)dn_i \tag{22}$$

which implies

$$\frac{dq_{bi}^*}{dn_i} = \frac{q_{bi}^* c''\left(\frac{1-n_i}{n_i}q_{bi}^*\right)}{n_i\left[(1-n_i)\,c''\left(\frac{1-n_i}{n_i}q_{bi}^*\right) - n_i u''(q_{bi}^*)\right]} > 0 \tag{23}$$

by concavity of $u(q)$ and convexity of $c(q)$. ■

This result shows that, when the local market i is thinner for buyers (that is n_i is smaller), market clearing condition implies a lower buyer's demand of the good. This is because, under price taking, the aggregate demand is equal to the aggregate production in equilibrium.

Next we establish a parallel result regarding the dynamics of prices in country i.

Proposition 2 *The country-i price of goods is decreasing in the country-i fraction of sellers.*

Proof. Substituting q_{si} into (10) from (15) and taking the differential yields

$$c''\left(\frac{1-n_i}{n_i}q_{bi}\right)\frac{1-n_i}{n_i}dq_{bi}-\frac{q_{bi}}{n_i^2}c''\left(\frac{1-n_i}{n_i}q_{bi}\right)dn_i=\phi dp_i \tag{24}$$

then differentiating (13) gets

$$u''(q_{bi})\,dq_{bi}=\phi dp_i \tag{25}$$

unless the country-i buyers' constraint is binding. Using (25) to eliminate dq_{bi} into (24) yields

$$c''\left(\frac{1-n_i}{n_i}q_{bi}\right)\frac{\phi(1-n_i)}{n_i u''(q_{bi})}dp_i-\frac{q_{bi}}{n_i^2}c''\left(\frac{1-n_i}{n_i}q_{bi}\right)dn_i=\phi dp_i$$

which implies

$$\frac{dp_i}{dn_i}=\frac{q_{bi}c''\left(\frac{1-n_i}{n_i}q_{bi}\right)u''(q_{bi})}{n_i\phi\left[(1-n_i)\,c''\left(\frac{1-n_i}{n_i}q_{bi}\right)-n_i u''(q_{bi})\right]}<0 \tag{26}$$

by concavity of $u(q)$ and convexity of $c(q)$. ■

Proposition 3 *Asymmetric fraction of sellers across countries is a necessary and sufficient condition for prices and the efficient quantity of good demanded by buyers to be asymmetric across local markets.*

Proof. (*Sufficiency.*) Assume the fraction of sellers is asymmetric across countries, i.e. $n_i \neq n_j$, $i \neq j$ where $i, j \in \{1, 2\}$. Then, directly from (23) and (26), this implies

$$q_{bi}^* \neq q_{bj}^*$$

and

$$p_i \neq p_j$$

respectively.

(*Necessity.*) Assume $q_{bi}^* \neq q_{bj}^*$ and $p_i \neq p_j$, $i \neq j$ where $i, j \in \{1, 2\}$. Then using (23) and (26) it holds that $n_i \neq n_j$. ■

5 Equilibria

This section characterizes equilibria in which efficiency is attained in either or both countries.

5.1 Global efficiency

Let the term global efficiency denote a stationary monetary equilibrium in which efficiency is attained in both countries, i.e. $q_{bi} = q_{bi}^*$ for any $i \in \{1, 2\}$.

We now state our first main result.

Proposition 4 *A global efficiency equilibrium is a necessary and sufficient condition for the marginal value of money to be equal to its marginal cost across countries.*

Proof. (*Sufficiency.*) Assume a global efficiency equilibrium holds, i.e. $q_{bi} = q_{bi}^*$ for any $i \in \{1, 2\}$. Then, expression (19) reduces to $V_i'(m_1) = \phi$ for any $i \in \{1, 2\}$.

(*Necessity.*) Assume the marginal value of money is equal to its marginal cost across countries, i.e. $V_i'(m_1) = \phi$ for any $i \in \{1, 2\}$. This is equivalent to the system of equations

$$\begin{cases} V_i'(m_1) = \phi \left\{ (1 - n_i) \left[\frac{u'(q_{bi})}{c'\left(\frac{1-n_i}{n_i} q_{bi}\right)} - 1 \right] + n_i \right\}, \\ u'(q_{bi}) = c'\left(\frac{1-n_i}{n_i} q_{bi}\right) \end{cases} \tag{27}$$

which, by virtue of (16)-(17), is satisfied if and only if $q_{bi} = q_{bi}^*$ for any $i \in \{1, 2\}$. ■

The meaning of Proposition 4 is the following. Under global efficiency, real balances are maximized across countries. This means that the country-i marginal value of taking an additional unit of money into the local market-i

$V_i'(m_1)$ is equal to its marginal cost (ϕ) which is a constant. So, the marginal value of money must be the same across local markets.

We now derive a sufficient and necessary condition for global efficiency.

Proposition 5 *The Friedman rule is a necessary and sufficient condition for global efficiency.*

Proof. (*Sufficiency.*) Assume that the economy is at the Friedman rule, i.e. $\gamma = \beta$. Using (20), this implies

$$(1 - n_i)\left[\frac{u'(q_{bi})}{c'\left(\frac{1-n_i}{n_i}q_{bi}\right)} - 1\right] = 0. \tag{28}$$

which implies

$$\frac{u'(q_{bi})}{c'\left(\frac{1-n_i}{n_i}q_{bi}\right)} = 1,$$

hence, using (17), $q_{bi} = q_{bi}^*$ for any $i \in \{1, 2\}$.

(*Necessity.*) Assume global efficiency, i.e. $q_{bi} = q_{bi}^*$ for any $i \in \{1, 2\}$. Replacing q_{bi} into (20) with q_{bi}^* and using (17) yields $\gamma = \beta$. ■

By Proposition 5, global efficiency is attained if and only if the money growth rate obeys the Friedman rule. The reason for this result is that, when agents are price takers and the Friedman rule is implemented, there is no hold-up problem, which makes the buyers' cash constraint non-binding across countries and so the quantity of good traded is efficient.

5.2 Local efficiency

Local efficiency is meant to be an equilibrium in which efficiency is attained in at least one country, i.e. $q_{bi} = q_{bi}^*$ for at least one $i \in \{1, 2\}$.

The next result establishes that the Friedman rule is necessary for local efficiency.

Proposition 6 *The Friedman rule is a necessary condition for local efficiency.*

Proof. Assume both that the Friedman rule is not implemented, $\gamma \neq \beta$, and local efficiency holds, i.e. $q_{bi} = q_{bi}^*$ for at least one $i \in \{1, 2\}$, let this be $i = 1$.

It follows from (17) that

$$\frac{u'\left(q_{b1}\right)}{c'\left(\frac{1-n_1}{n_1}q_{b1}\right)}=1, \tag{29}$$

which implies

$$\left(1-n_1\right)\left[\frac{u'\left(q_{b1}\right)}{c'\left(\frac{1-n_1}{n_1}q_{b1}\right)}-1\right]=0 \tag{30}$$

Substitution of (20) into (30) gives $\gamma-\beta=0$. This gives a contradiction, and shows that local efficiency implies the Friedman rule $\gamma=\beta$. ■

6 Conclusions

This paper analyzed the relationship between the Friedman rule and efficient allocations of resources within a new-generation, search-theoretic model of monetary union with divisibility and degenerate distribution of money holdings.

The main results are: *i)* the Friedman rule is a necessary and sufficient condition for global efficiency, and *ii)* the Friedman rule is necessary for local efficiency, as it is the unique policy rule that eliminates the hold-up problem in any local market. It is also shown that the efficient quantity demanded by a country-i buyer is increasing, and the country-i price of goods is decreasing, in the country-i fraction of sellers.

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