Coexistence of Monies as the Asymmetric Equilibrium of an Anonymous Game

(Very preliminary- please do not quote)

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Abstract

The present paper shows that the presence of a government committed to accepting legal tender ensures that legal tender coexists with either an international or a local currency, even when residents have identical preferences. This result is obtained as an asymmetric equilibrium of an anonymous game with atoms. Hence, monies coexistence may arise as a consequence of money demand equilibrium conditions, rather than strategic externalities associated with economic integration.

There is a symmetric equilibrium where everybody goes for legal tender only when it gives a payoff greater than the other money. The results are derived by constructing a game for the choice of UK residents between Pounds and Euros.

1 Introduction

The introduction of the European common currency puts both prospective EMU members and neighborhing countries in a peculiar situation. Will the Euro eventually become a currency common to UK and Poland etc. as

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well, regardless of these countries formally joining the EMU? If this happens, will the Euro be the unique circulating money or will it coexist with these countries' legal tender?

A distinct but related issue is that of dollarization. To what extent do residents of a country which monetary authority credibly commits to exchanging legal tender for US dollars continue to demand the national currency? It is expected that the preferability of the use of legal tender vanishes as the fraction of residents holding US dollars increases.

Another example is given by the choice between the use of cash versus credit cards. The advantages of credit cards increase as their use and acceptance spread over the community.

In all these instances, the distribution of choices among residents affects every single resident's choices. For example, the incentive to using the Euro to any UK resident is affected by the others residents' choices. Yet, this influence is not likely to take place at the individual level, rather it acts in an anonymous way through the distribution of choices over the entire UK population.

The object of the present paper is to study the effects of the government's commitment to accepting legal tender on private agents' choices as regards to the demand for legal tender in the presence of alternative monetary instruments.

Anonymous games (see [1]) provide a natural framework for the analysis of issues of demand for monetary instruments (see [3].)

In principle, the presence of the government imposes a departure from the anonymous game setting as its action cannot be taken as negligible by the other players. Still, in this study the non-negligible influence of the government's commitment is modeled by assuming a strictly positive probability of every agent meeting some other agent (namely the government) willing to exchange legal tender (even when it gives a payoff lower than the alternative monetary instrument). While this *tantamounts* to assuming a strictly positive frequency of private agents meeting with the public sector as [2] do, making this assumption permits to cast the analysis inside a parametrized family of standard anonymous games.

As is usual in monetary models, this model displays multiple equilibria: there exists an asymmetric equilibrium (i.e. individuals with same preferences play different actions) where legal tender coexists with an alternative money even when residents have identical preferences. A symmetric equilibrium where everybody goes for legal tender is selected when legal tender

gives the higher payoff. These possibilities are illustrated by constructing a game for the choice between Pounds and Euros.

No assumption on matching processes between pairs of agents, let alone non-uniform processes, is made. Rather, a foundation for the coexistence of monies is given that is independent of the existence of more than a single regional economy, but that relies only on the alternative money (which is thought of as an international currency) providing higher utility than what legal tender does. Utility from holding the alternative money can be seen as depending on a strategic externality similar to but different from the one Matsuyama et al. [3] postulate. This is related to how often a resident expects to meet another resident (and not a member of the other country as in Matsuyama et al. [3]) holding the other currency which is not legal tender (NLT), to the end of not diverting trade form within the community as a means to preserving the community members' welfare. This interpretation is supported by the examples of local communities that use own monies which are not legal tender (such as *Disney dollars*, the *Itacha*, NY, currency, and *Raam* in Vedic City.) Such a strategic externality makes the analysis independent of both the size of the two economies and their degree of integration. Hence, the coexistence of a *community money* and legal tender appears to be a phenomenon analogous to the coexistence of the Euro (that is not legal tender in the UK) and the Sterling Pound. In Matsuyama et al. [3] it is the strictly positive probability of a resident meeting a non-resident (who holds NLT) that associates the strategic externality from holding NLT with economic integration.

Hence, an important result of this paper is that coexistence of monies in the form of an asymmetric equilibrium obtains even if this probability is zero, showing how the strategic externality from holding NLT may arise from money demand equilibrium conditions, rather than from economic integration. Some evidence supporting this conclusion may be found in the goal of MCommunities which are usually meant to prevent wealth outflows from the area where the community resides. Historical examples support the view that the existence of the strategic externality may be independent of the degree of economic integration, even for *fiat* monies (which have no intrinsic value.)

This study is related to the literature on bimetallism, since the coexistence of monetary instruments is analyzed. It departs from that literature in that none of the coexisting monies is a commodity, nor any fixed exchange rate between monies is modeled. It is related to the literature on the *absence-of-double coincidence of needs* models of money, although the important issue

of the disparate short-run and long-run effects of changes in the quantity of money is not touched upon. Nor is examined the question of coexistence of money and assets with higher rates of return. Rather, the coexistence of different monies with different "utility returns" is modeled as a consequence of Cournot-Nash Equilibrium Distribution.

The scheme of the paper is the following: Section 2 puts forth the model. Section 3 characterizes monies demand as equilibrium distributions, and characterizes conditions for monies coexistence. Finally, Section 4 ends the paper with a summary of results.

2 The model

The analysis is cast inside a standard anonymous game. There are two types of agents: national residents and foreigners. Every national resident is assumed to make a dichotomic choice with regards to money holdings: she either holds the Pounds or Euros, not both simultaneously (a more general specification of the action set would only make the analysis less simple without affecting the result). Hence, actions are elements of the countable set $A = \{£, E\}$ where obviously $£$ stands for national currency and E stands for Euros. Since we are not assuming heterogeneity between either type of agent, ρ denotes the Dirac measure concentrated on the utility function $u \in U$. Hence, all residents are endowed with identical preferences and ρ is an atomic game (**anonymous matching requires non-atomicity**).

We capture the fact that Pounds are legal tender by assuming that the payoff that every agent gets from Pound holdings is independent of the distribution of the action $£$ among residents. If we denote by $\nu \in M(A)$ an element of the set of probability measures on A, then this assumption *tantamounts* to

$$u(£, \nu) = \varphi \tag{1}$$

where $\varphi \in (0, 1]$ is a constant. The assumption on φ is made in order to simplify the analysis. Obviously, utility from legal tender cannot be zero, apart from widespread examples of bankruptcy which will be examined later, while taking 1 as the upper bound is a convenient normalization. This hypothesis rules out the outcome where residents do not hold any legal tender.

On the other hand, the payoff from holding Euros is increasing in the

fraction of residents making the same choice

$$u(E, \nu) = \nu(E)$$

where $\nu(E) \in [0, 1]$. This is a convenient way to assume the existence of a strategic externality as in Matsuyama et al. [3]. We do not require it, though, to arise from either the degree of economic integration or the relative size of economies.

3 Equilibrium distribution demand of monies

We now characterize the demand for monies as the equilibrium distribution of the anonymous game outlined in the previous Section. To this end, we borrow from [1] and [4] the following

Definition 1 *A CNED for this game is a probability measure τ on $U \times A$ such that*
(i) $\tau_U = \rho$ and
(ii) $\tau(B(\tau)) = 1$,
where
$B(\tau) = \{(u, a) \in U \times A : u(a, \tau_A) \geq u(x, \tau_A) \forall x \in A\}$
and τ_U and τ_A are the marginals of τ on U and A respectively.

3.1 Symmetric and asymmetric equilibria

A CNED always exists by [1]. In this game there are two CNEDs. When Pounds bring the highest utlity, no resident holds Euros

Proposition 2 *(no coexistence of monies.) If $\varphi > \nu(E)$ then every resident holds legal tender.*

When utility from legal tender is higher than what Euros pay, a symmetric equilibrium is selected where all homogeneous players go for Pounds. If utility from holding Euros is strictly less than utility from legal tender, which is constant, everybody prefers to keep Pounds. Hence, the only way to adopt the Euro is to join the EMU.

The second equilibrium features coexistence of monies that are equally distributed among residents.

Proposition 3 *(coexistence of monies.) If* $\varphi \leq \nu(E)$ *then monies are equally distributed among residents.*

Proof. (this closely follows [4]) Assume the equilibrium distribution of E is less than φ. Then $\tau_A(E) < \varphi$. Hence, $u(£, \tau_A) > u(E, \tau_A)$ and (ii) implies $\tau(B(\tau)) = \tau(u, £) = 1$. But, this in turn implies $\tau_A(£) = 1$, so all residents go for Pounds. Conversely, assume the equilibrium distribution of E is greater than φ. Then $\tau_A(E) > \varphi$. Hence, $u(£, \tau_A) < u(E, \tau_A)$ which implies $\tau(B(\tau)) = \tau(u, E) = 1$. But, this in turn implies $\tau_A(E) = 1$, which cannot be[1]. It must be, then, that in equilibrium $\tau_A(E) = \tau_A(£) = \varphi$. But since τ_A is a probability measure on A, $\tau_A(E) + \tau_A(£) = 1$, whence $\varphi = \frac{1}{2}$. We have proved that in equilibrium the two currencies must be equally distributed among residents. ■

In this latter case, currencies cannot coexist in equilibrium. The reason for this is the following. While every player would go for Euros since they pay more, this cannot be an equilibrium unless Pounds are absolutely worthless. From the assumption $\varphi \in (0, 1]$, the distribution of legal tender cannot be zero, which would be a symmetric equilibrium. Hence, the equilibrium must necessarily be asymmetric, i.e. residents must play different actions even though they have identical preferences.

It is clear how the assumption $\varphi \in (0, 1]$ is crucial in determining the kind of possible equilibria. How can this be justified? Probably, it is conceivable that the probability of meeting another Pound holder is never zero inside the UK, while the probability of meeting another Euro holder can be zero. This is easily ensured by the presence of either a government or a monetary authority committed to accepting legal tender. This way, private agents know the probability of meeting another agent willing to accept Pounds is always strictly positive. Hence, if utility accrues from trade, utility from Pounds is bounded below by zero, but never attains its infimum, while utility from Euros does. In other words, the commitment of the government is sufficient to select an asymmetric equilibrium.

[1] If we had allowed for $\varphi \in [0, 1]$, then this outcome would be consistent with equilibrium in the form of a currency crisis where all residents go for Euros.

References

[1] Mas-Colell, A., 1984, On a theorem of Schmeidler, Journal of Mathematical Economics 13, 201-206

[2] Li, Y., and R. Wright, 1998, Government transaction policy, media of exchange, and prices, Journal of Economic Theory 81, 290-313

[3] Matsuyama, K., Kiyotaki N. and A. Matsui, 1993, Toward a theory of international currency, The Review of Economic Studies 60 (2), 283-307

[4] Rath, K. P., Sun Y. and S. Yamashige, 1995, The nonexistence of symmetric equilibria in anonymous games with compact action spaces, Journal of Mathematical Economics 24, 331-346

[5] Ravikumar, B. and N. Wallace, 2001, A Benefit of Uniform Currency, mimeo