



Working Papers in Economics and Finance

Endogenous heterogeneity

Pietro Senesi

September, 2019

ISBN 978-88-6991-151-4

Note: Working Papers in Economics and Finance are preliminary materials circulated to stimulate discussion and critical comment. The analysis and conclusions set forth and the views expressed therein are those of the authors and do not necessarily reflect any ideas nor any statements by any member of IRSAF. References in publications to working papers in Economics and Finance (other than acknowledgement) should be cleared with the author(s) to protect the tentative character of these papers.

Endogenous heterogeneity *

Pietro Senesi[†]

September, 2019

Abstract

This paper studies a Lagos and Wright economy with endogenous heterogeneity. In particular, the distribution of impatience (denoted by β) across agents converges pointwise to a degenerate distribution, the persistence of a δ -measure of agents with higher impatience, for some $\delta > 0$, notwithstanding. As a consequence, a non zero measure set of agents holding idle money balances exists in the absence of any randomness nor ex post heterogeneity. Hence, examples of LW economies where the efficiency of equilibrium allocations is improved by letting agents hold interest bearing assets are robust. The results also show that coexistence of money with bonds is not ruled out by pointwise convergence of the distribution of money over the set of agents to a constant function. More exactly, the distribution of money may converge pointwise but not uniformly. . .

Keywords: money, search, bonds

JEL Classification: E40, H20, H63

* . . .

[†]Department of Human and Social Sciences, University of Naples “L’Orientale”, Largo S. Giovanni Maggiore, 30 - 80134 Naples, Italy; email: psenesi@unior.it, Phone: +39 081 6909482, Fax: +39 081 6909442

1 Introduction

...

ACP define a *bilateral matching rule* and its *exhaustiveness* in terms of set theoretic concepts.

The first result they give is that of existence of *three pairwise disjoint sets* brought about by any bilateral matching rule. One of the three subsets sees agents matched with themselves. This is a set of fixed points. Another sees agents matched with the remaining set. The latter two sets have the same cardinality.

A finite set with an odd number of agents does not admit any exhaustive matching. A non-empty compact convex subset of a Hausdorff locally convex space does not admit any continuous exhaustive bilateral matching rule (it's unclear to me why.)

They go on defining a matching process, i.e. an iteration of the matching technology over time, and assume it is exhaustive in the sense that it necessarily matches every agent to someone else at every time $t \geq 1$, with $t \in \mathbb{N}$.

Hence, ACP construct a mapping between properties of the *matching process* and the *degree of informational openness* (i.e. the *degree of anonymity*) that are consistent with a physical description of the environment (which description?)

The fact that ACP work focuses on *exogenous matching* and abstracts from its *allocative effects* makes it more closely related to the strand of literature aimed at building mathematical foundations for *random matching* models with *countable* or *uncountable* populations. Such models are directed at obtaining a *law of large numbers* for random pairwise matching.

ACP remove any stochastic elements.

Ex-post inefficiency with some agents holding idle monetary balances and others being cash constrained is modeled as a consequence of trading shocks in random matching monetary models (see ...)

Accordingly, the existence of a credit market that reallocates money across agents is desirable but has no foundation in the absence of random shocks that hit agents and create a non-uniform distribution of impatience.

The present paper illustrates a LW economy where heterogeneity across agents in terms of impatience is the result of a deterministic subdivision of the agents set, with no random shock occurring. In such an economy, a credit market improves the allocation even though the distribution of impatience converges pointwise to a constant function. The credit market is essential because there exists a non-zero measure subset of most-impatient people in every equilibrium. In addition, every agent is bound to permanently become a low impatience agent, and no agent will ever be borrowing more than once. Hence, the credit market is essential even if there is no role for a record keeping technology.

[11] analyze credit sustainability when agents are matched repeatedly.

The paper is organized as follows. Section 2 describes the basic framework and the agents' decision problem. Values are characterized in Section 3. Section 4 states the results. The Conclusions end the paper.

2 Main result

Theorem 1 *There is a subset of $[0, 1]$ with strictly positive countably additive and translation invariant measure with higher impatience.*

Proof. Associate a rational number $r_t \in [0, 1)$ with each $t \in \mathbb{N}$. Let $2^{-t-1} \geq i > 2^{-t}$ and define $\beta_i(x) = \tilde{\beta} > \beta$ if $x \in P_t$ and $x = 2^{t+1}i - 1$, or $\beta_i(x) = \beta$ if $x \in [0, 1) \sim P_t$.

Then $\langle \beta_i \rangle$ is a sequence of measurable real valued functions on $[0, 1]$ such that for each x , $\lim_{i \rightarrow 0} \beta_i(x) = \beta$ but for some $\alpha > 0$ it holds that $m^*\{x : \beta_i(x) > \beta\} > \alpha$. ■

Remark. The P_t 's are pairwise disjoint, so that no agent becomes impatient again.

3 The environment

The framework of analysis is a modification of LW suited for the characterization of equilibria that feature an endogenous nominal interest rate.

Time is indexed by $t \in \mathbb{N}$.

In each period t two markets open sequentially. The first market to open is decentralized (DM), the second market is centralized (CM).

Agents

There is a closed unit interval $[0, 1]$ of infinitely-lived agents, so that every single agent is zero-measure. Thus, the chances of two single agents meeting are zero.

Buyers in the DM are *anonymous*.

Consequently, trade credit cannot occur. Transactions are subject to a *quid pro quo* restriction that allows money to play a role as a medium of exchange (Kocherlakota [15] and Wallace [24]).

When an agent comes across a good she demands, the same agent bargains with another agent (from a zero measure set of shop owners, e.g. the Cantor ternary set) in order to determine the terms of trade. The terms of trade depend on the distribution of portfolios across shop owners that sell specialized goods. Hence, the distribution of portfolios across agents induces a distribution of portfolios across goods for sale. This distribution is a step function with constant values on each α -measure set of goods delivered by the same seller. This is equivalent to having an α -measure of appropriate sellers for every buyer.

Special goods

A variety of goods is produced in the DM. Agents specialize in the production of goods other from those they consume, so that autarky cannot be an equilibrium.

Specification A.

Assumption A.1) Every agent specializes in the production of a single good g_i that is demanded by a zero-measure (e.g. countable) set agents. For this to be the case, it suffices to impose that each good is demanded by at most two agents.

Accordingly, the chances of selling the produced good are zero, though the producer may even match a countable infinity of buyers.

As an example, let the set of specialized goods be G . Then, every agent $i \in [0, 1]$ demands a variety of goods $G_i \subset G$ with G_i having α -measure. $G_i \cap G_j$ is nonempty with zero measure for every $i \neq j$, i.e. tastes across agents differ almost everywhere. This implies that there are no goods demanded by a nonzero measure set of agents. It also implies that if any set of goods are demanded by a nonzero measure set of agents, then such a set of goods must be zero measure. The set of buyers appropriate for seller i is denoted by B_i . Hence, *a1*) the measure of buyers appropriate for each seller $i \in [0, 1]$ is $\mu = 0$, while *a2*) the measure of sellers appropriate for each buyer $i \in [0, 1]$ is α .

a1) seems to capture Marx's setting where producers demand money to buy capital, a zero measure set of goods.

Does *a1*) imply that sellers in the DM are elements of a set distinct from the set of buyers (with both selling labor in the CM)?

Summing up, this specification says that the measure of purchases in the DM is $\alpha > 0$ while the measure of sales is zero.

For example, this is the case if most agents (an α -measure set) work only in the CM and buy specialized goods (produced by a zero measure set of agents) in the DM (once a week) and general goods in the CM.

Think of people buying goods from few producers on the internet . . . think of a village fair where lots of people go for a walk and buy from a small number of sellers . . .

Is this like people (it'd be better with like tastes) going to malls, with a very small number of people demanding the same particular good?

Portfolios can also be seen as distributed over special goods (in addition to being distributed over agents), in the sense that portfolios are constant over goods that are offered by the same shop owner. So the distribution of portfolios over special goods may be a step function.

Every agent demands an α -measure set of goods, and meets goods rather than sellers. This can be seen as representative of the situation where agents

go to huge mall where lots of goods are offered by a relatively little number of shop owners. In such a situation, the chances of a buyer coming across goods she demands are not negligible, while the chances of a particular seller being matched to a particular buyer are considerably fewer.

Specification B.

Assumption B.1) The measure of shop owners is zero.

Assumption B.2) Shop owners demand goods from a zero measure set (contrasted with other agents who demand an α -measure set of goods.)

Either B.1 and B.2 jointly, or

Assumption B.3) $m(G_i \cap G_j) = 0$ for every $i \neq j$,

along with

Assumption B.4) $m(G_i) = \alpha$ for every $i \in [0, 1]$,

imply:

i) the measure of sellers meeting appropriate goods is zero, and *ii*) the measure of buyers meeting appropriate goods is $\int_{[0,1]} \alpha = \alpha$.

Under the above assumptions, the measure of goods is $\sum_{i \in [0,1]} \alpha = \infty$, while in LW the measure of goods is 1.

This raises the question of *where do those goods that are not produced by any agent come from?* $[\bigcup_{i \in [0,1]} G_i] \sim [0, 1]$ is a set of goods that exist in nature (are primitive) and are owned by agents, e.g. different types of labor abilities. *Does it imply that the measure of shop producers-sellers be 1? Is the Cantor ternary set (it has measure 0) an admissible counterexample?*

The convenient feature is that value functions are considerably simplified and the constraints on numerical simulations parameterized by α is less stringent (e.g. LW get an upper bound of 0.5 on σ , the money velocity in the DM, which is related to α here).

It turns out that while the value of σ used in simulation by LW made no difference, it does when it comes to simulate the nominal interest rate, i.e. the nominal interest rate is very sensitive to α .

Special goods are non-storable and perish at the end of the DM, so that the only assets that can be carried onto the CM are money and bonds.

The evolution of agents portfolios

The distribution of assets holdings F_t changes as a consequence of agents trading.

An agent entering the DM with m_t exits with $m_t + d_t(\tilde{m}_t, m_t)$ in case of a single coincidence sale, and exits with $m_t - d_t(m_t, \tilde{m}_t)$ in case of a single coincidence purchase, with

As a result,

$$d : \mathbb{R}_+^2 \times \mathbb{N} \rightarrow \mathbb{R}_+ \quad (1)$$

maps F_t to F_{t+} .

An agent entering the CM with m_{t+} chooses $(m')(m_{t+}) = m_{t+1}$.

Agents trade assets in the CM in such a way that m_{t+1} is independent of m_{t+} , i.e. the value of portfolios does not matter, and (m') is a constant.

This maps F_{t+} into F_t again.

Preferences in the DM

Agents enjoy utility $u(q)$ from q consumption in the DM, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$.

Furthermore, the elasticity of utility $\eta(q) = qu'(q)/u(q)$ is bounded by assumption.

Producers incur a utility cost (a disutility) $c(q)$ from producing q units of output with $c'(q) > 0$ and $c''(q) \geq 0$.

Let q^* denote the solution to $u'(q^*) = c'(q^*)$.

Preferences in the CM

A single good is produced in the CM instead.

In the CM all agents consume and produce, enjoying utility $U(x)$ from x units of consumption, with $U'(x) > 0$, $U'(0) = \infty$, $U'(\infty) = 0$ and $U''(x) \leq 0$.

The same consumption can be produced from labor by each agent using a linear technology.

This implies that no wealth effects drive demand for money in the CM. Hence, money demand is also independent of trading histories.

Agents discount only between time t -CM and time $t + 1$ -DM.

This is not restrictive since as in Rocheteau and Wright [21] all that matters is the total discounting between successive periods.

Money supply

It is assumed a central bank exists that controls the money supply at time t , $M_t > 0$.

Money supply transforms according to $M_t = \gamma M_{t-1}$. $\gamma \in \mathbb{R}_+$ is a constant and new money is injected, or withdrawn if $\gamma < 1$, through transfers $\pi M_{t-1} = (\gamma - 1)M_{t-1}$ to agents **which types? any?**.

Money transfers are lump-sum (i.e. they do not depend on agents' behavior). We restrict attention to policies where $\gamma \geq \beta$, with $\beta \in (0, 1)$ denoting the discount factor.

Agents receive lump-sum money transfers π_b at the opening of DM trade.

Let $\pi_b M_{t-1} = \pi M_{t-1} / (1 - n)$ be the per agent money transfer.

The timing of events is shown in Figure 1. **Make it!**

Stationary equilibria

We study steady state equilibria, where aggregate real money balances are constant.

We refer to this as a stationary equilibrium

$$\phi M = \phi_{-1} M_{-1} \tag{2}$$

which implies that $\phi_{-1} / \phi = M / M_{-1} = \gamma$.

The Fisher equation does not necessarily hold, hence the equivalence of either setting the nominal interest rate or the inflation rate is not granted.

In period t , let $\phi_t = 1/P_t$ denote the real price of money and P_t the price of goods in the CM.

4 Values

Under stationary equilibrium the only source of uncertainty comes from random matching.

Aggregate variables enter individual maximization problems as fixed parameters.

Agents decisions are then implied by (common) VFs with money m as the only argument.

Let $V(m_t)$ denote the expected value from trading in the DM with m_t money balances.

Let $W(m_{t+})$ denote the expected value from entering the CM with m_{t+} units of money.

It is convenient to sequentially characterize equilibria within a single period starting from the CM.

Centralized market max problems

In the CM agents produce h units of good using h hours of labor, consume x , and adjust their money balances.

Bonds do not mature and cannot be exchanged among agents in the CM.

The real wage per hour is normalized to one.

Discounting explicitly appears in values, W s, calculated in CMs as they include next period's values, V_{+1} s.

4.1 Representative agent's CM problem

There is no dependence of either V or W on t . The notation m_{t+1} stands for money holdings carried onto next DM.

The representative agent's problem at the beginning of the CM

$$W(m_{t+}) = \max_{x, h, m_{t+1}} [U(x) - h + \beta V(m_{t+1},)] \quad (3)$$

such that

$$-h = -x + \phi(m_{t+} - m_{t+1}) \quad (4)$$

with $x \in \mathbb{R}_+$, $h, 0 \in H$ a connected closed and bounded subset of \mathbb{R}_+ , and $m_{t+1} \in \mathbb{R}_+$ denoting the money taken into period $t + 1$.

For money demand to be degenerate in the CM, utility from either labor supply or consumption must be linear. Following LW, it is assumed that utility is a linear function of labor supply, h .

Eliminate $-h$ from (3) using (4) and get

$$W(m_{t+}) = \max_{x, m_{t+1}} [U(x) - x + \phi_t(m_{t+} - m_{t+1}) + \beta V(m_{t+1})] \quad (5)$$

or

$$W(m_{t+}) = \phi_t m_{t+} + \max_{x, m_{t+1}} [U(x) - x - \phi_t m_{t+1} + \beta V(m_{t+1})] \quad (6)$$

where

$$V(m_{t+1}, 0) \quad (7)$$

is utility from buying with money.

The first order conditions with respect to x and m_{t+1} are

$$U'(x^*) = 1 \quad (8)$$

$$\beta V_{m_{t+1}}(m_{t+1}) = \phi_t \quad (9)$$

where the $\beta V_{m_{t+1}}(m_{t+1})$ is the seller's marginal benefit of taking money out of the CM and ϕ is its marginal cost.

Equation (8) characterizes the optimal consumption level x^* .

Equation (9) shows that m_{t+1} is independent of m_t , i.e. the distribution of money holdings across sellers is degenerate at the beginning of the next period because the quasi-linearity assumption in (3) **check reference** rules out any wealth effect on money demand in the CM.????

Agents who bring too much cash into the CM spend some buying goods, while those carrying too little sell goods.

Envelopes and no arbitrage

Rather, agents adjust money holdings in the CM so as to exploit arbitrage opportunities as below ???

Equations (6) and (??) imply the envelope conditions

$$W_{m_{t+}} = \phi_t \quad (10)$$

4.2 DMs

In the DM agents are allowed to barter, exchange specialized goods for money, and exchange specialized goods for bonds.

Let q_b and q_s denote the quantities consumed by a buyer and produced by a seller trading in the DM, respectively.

Agents may not find it optimal to carry entire portfolios to the market as this may reduce bargaining power, but this possibility will not be considered in what follows to simplify the analysis.

Let p be the nominal price of goods in the DM.

As anticipated, an agent carrying the portfolio m_t to the DM exits with $m_t + d_t(\tilde{m}_t, m_t)$ in case of a single coincidence sale of the quantity $q_t(\tilde{m}_t, m_t)$ to a buyer carrying the portfolio (\tilde{m}_t) , and exits with $(m_t) - d_t(m_t, \tilde{m}_t)$ in case of a single coincidence purchase of the quantity $q_t(m_t, \tilde{m}_t)$ from a seller carrying the portfolio (\tilde{m}_t) .

Assume agents carry the entire portfolios they own to the DM, and denote by $V(m_t)$ the value of entering the DM with portfolio m_t .

Then, under *Specification A*, each agent $i \in [0, 1]$ chooses a portfolio so as to maximize

$$\int_{G_i} \{u[q] + W - d\} + \int_{B_i} \{-\nu[q] + W + d\} \quad (11)$$

Hence, the value function can be written as

$$\begin{aligned} V(m_t) = & \\ & \max_{m_t} \left\{ \alpha \int \{u[q_t(m_t, \tilde{m}_t)] + W[m_t - d_t(m_t, \tilde{m}_t)]\} dF_t(\tilde{m}_t) \right. \\ & + \mu \int \{-\nu[q_t(m_t, \tilde{m}_t)] + W[(m_t) + d_t(\tilde{m}_t, m_t)]\} dF_t(\tilde{m}_t, \tilde{b}_t) \\ & \left. + (1 - \alpha)W(m_t) \right\} \quad (12) \end{aligned}$$

where $F_t(\tilde{m}_t)$ denotes the (induced by sellers specialization) distribution of portfolios across the α -measure set of goods appropriate for agent (buyer) i , and $\mu = 0$ is the measure of buyers appropriate for agent i .

The first term is . . .

Assume that

$$W[(m_t) - d_t(m_t, \tilde{m}_t)] = W[(m_t)] - \phi d_t(m_t, \tilde{m}_t) \quad (13)$$

and

$$W[(m_t) + d_t(\tilde{m}_t, m_t)] = W[(m_t)] + \phi d_t(\tilde{m}_t, m_t) \quad (14)$$

Then (12) reduces to the simpler form

$$\begin{aligned} V(m_t) = & \\ & \max_{m_t} \{ \alpha \int \{ u[q_t(m_t, \tilde{m}_t)] - \phi d_t(m_t, \tilde{m}_t) \} dF_t(\tilde{m}_t) \\ & + W(m_t) \} \end{aligned} \quad (15)$$

LW get rid of this integral because the money the buyer pays is independent of the seller's money holdings (this makes the distribution of money irrelevant but may not be realistic in some cases.)

In other words, the zero measure of sellers jointly with the independence of the money payment on the seller's money holdings (and, different from LW, no chances of bartering) give the following value

$$\begin{aligned} V(m_t) = & \\ & \max_{m_t} \{ \alpha \{ u[q_t(m_t, \tilde{m}_t)] - \phi d_t(m_t, \tilde{m}_t) \} \\ & + W(m_t) \} \end{aligned} \quad (16)$$

4.2.1 Bargaining

In the Nash problem

$$\max_{q, d_t(m_t, \tilde{m}_t), d_t(\tilde{m}_t, m_t)} [u(q) - \phi d_t(m_t, \tilde{m}_t)]^\theta [-\nu(q) + \phi d_t(\tilde{m}_t, m_t)]^{1-\theta} \quad (17)$$

what the buyer pays is equal to what the seller gets, so that

$$\max_{q, d_t(m_t, \tilde{m}_t)} [u(q) - \phi d_t(m_t, \tilde{m}_t)]^\theta [-\nu(q) + \phi d_t(m_t, \tilde{m}_t)]^{1-\theta} \quad (18)$$

Assume $\theta = 1$ so that the above problem simplifies to

$$\max_{q, d_t(m_t, \tilde{m}_t)} [u(q) - \phi d_t(m_t, \tilde{m}_t)] \quad (19)$$

The solution depends both on the degree of mildness of disutility from labor and on whether the budget constraint binds.

If disutility is mild enough and the constraint binds the solution is characterized by

$$u'(q) > 0, \lambda_b > 0 \quad (20)$$

with λ_b denoting the Lagrange multiplier on the buyer's budget constraint, and trade is inefficient. Otherwise, $u'(q)$ must equate the utility cost of giving money up.

Is this expressed by

$$u'(q) = \phi \quad (21)$$

?

If the answer is yes, assume $u(q) = \ln(q)$ so that

$$q^* = \phi^{-1} \quad (22)$$

so that the terms of trade are (ϕ^{-1}, ϕ) , i.e. the quantity ϕ^{-1} is exchanged at the utility price ϕ of a unit of money (equivalently, for a unit of money).

If there is no disutility from labor, then the quantity produced and exchanged is efficient, and so $d = m^* = m_t$, where m^* is the least amount of assets (money and bonds) sufficient to induce the seller to produce and offer the quantity q^* .

If the answer is no, consumer equilibrium is given by

$$u'(q) = \phi p \quad (23)$$

Assuming $p = 1$, it follows that

$$q^* = \phi^{-1} = 1 \quad (24)$$

and the terms of trade reduce to $(q^*, p) = (1, 1)$, i.e. the quantity 1 is exchanged at the utility price 1 of a unit of money (equivalently, for a unit of money).

Again, if there is no disutility from labor, then the quantity produced and exchanged is efficient, and so $d = m^* = m_t$, where m^* is the least amount of money sufficient to induce the seller to produce and offer the quantity $q^* = 1$.

The value (16) becomes

$$V(m_t) = \alpha(\ln[1] - 1) + W(m_{t+}) \quad (25)$$

or

$$V(m_t) = -\alpha + W(m_{t+}) \quad (26)$$

Assume linear utility in the CM, $U(x) = x$, and use (5) to get

$$V(m_t) = -\alpha + \max_{(m_{t+1})} \{\phi_t m_{t+} - \phi_t m_{t+1} + \beta V(m_{t+1})\} \quad (27)$$

where $\phi(m_{t+} - m_{t+1})$ is the value of money inside next period portfolio net of the cost of acquiring it, and (??) implies maximization only w.r.t. m_{t+1} .

The term $\nu_0(s)$ in LW2002 is zero here.

Hence,

$$V(m_t) = -\alpha + \max_{m_{t+1}} \{\phi_t m_{t+} - \phi_t m_{t+1} + \beta V(m_{t+1})\} \quad (28)$$

Repeated substitution gives

$$V(m_t) = -\alpha + \phi_t m_{t+} + \sum_{j=t}^{\infty} \beta^{j-t} \max_{m_{j+1}} \{-\phi_j m_{j+1} + \beta[\nu_{j+1}(m_{j+1}) + \phi_{j+1} m_{j+1}]\} \quad (29)$$

or

$$V(m_t) = -\alpha + \phi_t m_{t+} + \sum_{j=t}^{\infty} \beta^{j-t} \max_{m_{j+1}} \{m_{j+1}(\beta\phi_{j+1} - \phi_j)\} \quad (30)$$

This is simpler than LW as we got rid of their ν_{t+1} which depended on F_{t+1} , $\nu_{t+1}(F_{t+1})$. So there is no dependence of the sequence of m'_j s on F_{t+1} (LW say it only influence the intercept of the VF and not the m_{t+1}).

$V(m_t)$ is linear in m_{j+1} . If $\beta\phi_{j+1} - \phi_j > 0$, i.e. $\beta\phi_{j+1} > \phi_j$ or $\frac{\phi_{j+1}}{\phi_j} > \frac{1}{\beta}$, then there is no solution to the problem of choosing m_{t+1} ... why? Is the no-arbitrage condition of help?

Looks like equilibrium requires $\beta\phi_{j+1} < \phi_j$ as in LW. Does it imply that the optimal m_{t+1} is zero and agents only hold bonds? Yes it implies $m_{t+1}^* = 0$. Don't know if bonds are positive.

Notice that LW characterize monetary equilibrium by any path for $\{q_{t+1}\}$ satisfying $m_{t+1} < m_{t+1}^*$ (LW, p. 472).

If $\phi_{j+1} = \phi_j$ as, given b_{t+} , eq. (??) suggests (this holds also because of Lemma 3 in LW2002) ... then $\beta\phi_{j+1} < \phi_j$ holds and $m_{t+1}^* = 0$. If so, then $V(m_t, b_t) = -\alpha + \phi_t m_{t+}$.

Whose hands the money spent in the CM goes? As nobody is carrying any money into next DM, everybody must be spending the whole of money holdings (including money injection which takes place at the beginning of the CM).

Hence, agents find it optimal to maximize the value of their portfolios and the portfolio constraint always binds. As a consequence, neither the equilibrium demand for money nor for bonds can be zero. Hence, money and bonds coexist? Even though agents try to substitute money for bonds ...

...

General bargaining solution

The general solution characterized by LW consists of the seller spending the $d_t(m, \tilde{m}) = \min(m_t, m^*)$.

If $m_t = \min(m_t, m^*)$ then the buyer gets $q_t(m, \tilde{m}) = \tilde{q}_t(m) \leq q^*$.

If $m^* = \min(m_t, m^*)$ the cash constraint is not binding and the buyer gets $q_t(m, \tilde{m}) = q^*$ (and eventually disposes or what of excess money holdings?)

(This consists of either the seller exchanging all of his money holdings ($d_t(m, \tilde{m}) = m$) for a quantity weakly less than the efficient level $q_t(m, \tilde{m}) = \tilde{q}_t(m) \leq q^*$ (if $m_t \leq m^*$ with m^* denoting the least amount of money sufficient to buy q^*), or the buyer giving all of her money holdings ($d_t(m, \tilde{m}) = m < m_t^*$) up for a lesser quantity ($m_t < m^*$ and the budget constraint is binding.))

Hence, in LW the solution to the bargaining problem only depends on the buyer's money holdings m_t and I cannot get any discount from a starving seller!

Remark. The LW solution to the bargaining problem \Rightarrow the buyer cannot get any utility from money in excess of $m^* \Rightarrow \nu'_{t+1}(m_{t+1}) = 0$ for all $m_{t+1} \geq m_{t+1}^* \Rightarrow$ any equilibrium must satisfy $\phi_t \geq \beta\phi_{t+1}$.

If bonds allow buyers to get utility from that extra cash, will the minimum inflation rate consistent with equilibrium still be the Freidman rule? see LW p. 471.

Let agents with more than m^ lend to those with less. This should be welfare improving as more people consume closer to (at) the efficient level. There is a role for credit with no banks, no government, and no ex-post heterogeneity!*

Is $m_{t+1}^* = m^*$?

The bargaining solution can be used to simplify the value function (12).

...

5 Conclusions

To be added.

References

- [1] Andolfatto, D. 2006, "Revisiting the Legal Restrictions Hypothesis," mimeo
- [2] Berentsen, A. Camera, G. and Waller, C. 2007, "Money, Credit and Banking," *Journal of Economic Theory* **135**, 171-195
- [3] Berentsen, A. Camera, G. and Waller, C. 2005, "The Distribution of Money Balances and the Non-Neutrality of Money," *International Economic Review* **46**, 465-487
- [4] Berentsen, A. Rocheteau, G. and Shi, S. 2007, "Friedman Meets Hosios: Efficiency in Search Models of Money," *Economic Journal* **117**, 174-195
- [5] Berentsen, A. and Waller, C. 2005, "Optimal Stabilization Policy with Flexible Prices," mimeo
- [6] Berentsen, A. and Waller, C. 2007, "The Societal Benefits of Outside Bonds Versus Inside Bonds," mimeo
- [7] Boel, P. and Camera, G. 2006, "Efficient Monetary Allocations and the Illiquidity of Bonds," *Journal of Monetary Economics* **53**, 1693-1715
- [8] Bewley, T. 1980, "The Optimum Quantity of of Money," in J.H. Karaken and N. Wallace, eds., *Models of Monetary Economics*, FRB Minneapolis, 162-210
- [9] Cavalcanti, R. and Wallace, N. 1999, "Inside and Outside Money as Alternative Media of Exchange," *Review of Economic Dynamics* **2**, 104-136
- [10] Cavalcanti, R. Erosa, A. and Temzelides, T. 1999, "Private Money and Reserve Management in a Random Matching Model," *Journal of Political Economy* **107**, 929-945
- [11] Corbae, D. and Ritter, J. 2004, Credit without memory, *Economic Theory* **24**, 933-951

- [12] Green, E. and Zhou, R. 2005, "Money as a Mechanism in a Bewley Economy," *International Economic Review* **46**, 351-371
- [13] He, P. Huang, L. and Wright, R. 2005, "Money and Banking in a Search Equilibrium," *International Economic Review* **46**, 637-670
- [14] Kehoe, T.J. Levine, D.K. and Woodford, M. 1990, "The Optimum Quantity of Money Revisited," FRB Minneapolis *Working Paper* no. 404
- [15] Kocherlakota, N. 1998, "Money is Memory," *Journal of Economic Theory* **81**, 232-251
- [16] Kocherlakota, N. 2003, "Social Benefits of Illiquid Bonds," *Journal of Economic Theory* **108**, 179-193
- [17] Lagos, R. and Wright, R. 2005, "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* **113**, 463-484
- [18] Levine, D. 1991, "Asset Trading Mechanisms and Expansionary Policy," *Journal of Economic Theory* **54**, 148-164
- [19] Li, V. 1995, "The Optimal Taxation of Fiat Money in Search Equilibrium," *International Economic Review* **36**, 927-942
- [20] Li, V. 1997, "The Efficiency of Monetary Exchange in Search Equilibrium," *Journal of Money, Credit and Banking* **29**, 61-72
- [21] Rocheteau, G. and Wright, R. 2005, "Money in Search Equilibrium, in Competitive Equilibrium and in Competitive Search Equilibrium," *Econometrica* **73**, 175-202
- [22] Shi, S. 1996, "Credit and Money in a Search Model with Divisible Commodities," *Review of Economic Studies* **63**, 627-652
- [23] Shi, S. 1997, "A Divisible Search Model of Fiat Money," *Econometrica* **65**, 75-102

- [24] Wallace, N. 2001, "Whither Monetary Economics?" *International Economic Review* **42**, 847-869
- [25] Zhu, T. and Wallace, N. 2007, "Pairwise Trade and Coexistence of Money and Higher-Return Assets," *Journal of Economic Theory* **133**, 524-535