Emergence, Evolution and Maintenance of Communication Conventions

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May 12, 2007

Abstract: We report the results of a simulation investigating in which circumstances communicative conventions emerge and can be maintained within a population of agents under different selective pressures. We will give a game theoretical analysis of few relevant cases. We will then report our empirical results of the error threshold phenomenon analytically proved in Nowak et al. [2].

Introduction

Communication systems can be characterized as a mapping from a set of meanings to a set of signals. We attempt in this paper to set a general framework to describe under which circumstances an optimal convention (or Saussurean Communication) can be established within a population of individuals.

The model

We will consider a scenario of 400 communicative individuals in an unbounded, torus shaped space. This scenario is similar to the one described in Oliphant 1994 [3].
The population of agents shares a finite set of signals used to convey a corresponding amount of shared meanings. Each individual is born with a transmitting system specifying which signal is associated with each meaning and a receiving system mapping back each symbol to a specific meaning. We will take into consideration the very general case in which the receiving system doesn't necessary mirror the transmitting system.
Each agent has 8 neighbours and goes through 5 life stages; after the fifth the first follows. In each stage the agents try to establish 8 random communications with their neighbours. In every communication two agents act alternately as speakers and listeners. The speaker picks one random meaning out of the meaning space and sends the associated signal to the hearer (according to its transmitting system). The hearer uses its receiving system to find the meaning associated with the received signal. If this meaning is the same as originally chosen, the communication is said to be successful.

During the fist stage, an individual selects a teacher from the set of its neighbours according to the communication success (fitness) they have accumulated during the previous life stage (a neighbour with double fitness has double probability to be selected). The newborn agent then acquires with a certain precision the communication system of its teacher: each mapping in the teacher's communication system is learned with probability $1 - \mu$ and it is randomly chosen with probability μ (mutation rate).

The simulation

In order to start with the simulations we need to take into consideration several parameters. Each set of parameters corresponds to a specific communication scheme. The aim of this study is therefore to understand for which communication schemes a Saussurean Communication can emerge and can be maintained within the population. We can describe the parameters as follows:

N: number of meanings used by the agents, equivalent to the number of forms (symbols) they are able to express.

μ**:** the mutation rate characterizing the acquisition phase.

Initialization : whether in the starting point of the simulation each agent is initialized with a random communication system (*random start*) or with a unique optimal communication system (*optimal start*).

Payoff Matrix: when communicating to its neighbours, each agent cumulates a transmission and reception success rate. The payoff matrix specifies how the fitness function is calculated in respect to these two values. Two general cases are taken into consideration: the case where the fitness is the average of the two values (*both speakers and listeners are rewarded*), and the case where the fitness coincides with the reception success rate (*only listeners rewarded*).

Spatial organization: whether the agents keep the same position during the life cycle (*spatially organized*) or are randomly displaced at each iteration (*non spatially organized*).

Iterative: this is an optional mode; when active each agent has two possible communication systems: in each communication attempt each agents uses the first communication system if the previous reception was successful, the second if unsuccessful. Each communication between two agents is iterated 16 times.

Oliphant's Results

We will run here the 4 simulations reported in Oliphant. In all the cases we have N = 2 and a random initialization startup: each agent is initialized with a random communication system and a random age (life stage).

Simulation 1

In the very first case we consider the default setting: only listeners are rewarded in a non-spatially organized population. Under these parameters Saussurean Communication can emerge but is not stable. Figure 1 shows a snapshot of a simulation where $\mu = 0.02$.

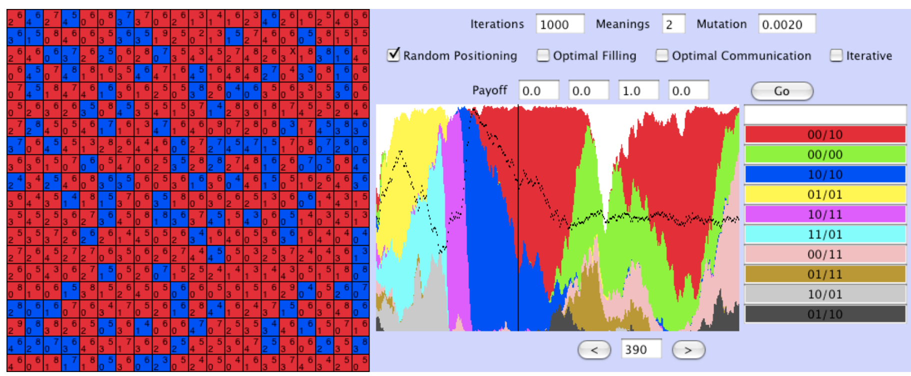

Figure 1: snapshot simulation 1

The left side panel represents the population of agents. The color of each square characterizes the communication system of the associated agent. The bottom left number is the age and the top right number is the fitness rounded and multiplied by 10 (X stands for 10). The right side panel represents the frequency of each colored communication system for each iteration. The black line specifies the level of the average fitness of the entire population.

We notice how the blue optimal communication system (10/10) emerges around iteration 300 but it is suddenly replaced by the red population (00/10) which has the same payoff of the blue agents since it has the same receiving system (only listeners are rewarded).

That no communication system is stable with these parameters is clear from Table 1: taken any communication system X, there is always a different communication system Y such that $F(X,X) \leq F(Y,X)$ and if it is equal $F(X,Y) \not> F(Y,Y)$, where F is the payoff function. Hence, there exists no "evolutionary stable strategy" [1]. In particular, in a population where any of the two optimal communication systems (01/01 or 10/10) is dominating, three other suboptimal communication systems have the chance to invade: they perform equally good when communicating with the dominants and no worse (equal) when communicating among themselves.

F	00 00	00 01	00 10	00 11	01 00	**01** **01**	01 10	01 11	10 00	10 01	**10** **10**	10 11	11 00	11 01	11 10	11 11
00/00	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
00/01	.5	**.5**	.5	.5	1	**1**	1	1	0	0	0	0	.5	.5	.5	.5
00/10	.5	.5	**.5**	.5	0	0	0	0	1	1	**1**	1	.5	.5	.5	.5
00/11	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
01/00	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
01/01	.5	**.5**	.5	.5	1	**1**	1	1	0	**0**	0	0	.5	**.5**	.5	.5
01/10	.5	.5	.5	.5	0	0	**0**	0	1	1	**1**	1	.5	.5	.5	.5
01/11	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
10/00	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
10/01	.5	.5	.5	.5	1	1	1	1	0	0	0	0	.5	.5	.5	.5
10/10	.5	.5	**.5**	.5	0	0	**0**	0	1	1	**1**	1	.5	.5	**.5**	.5
10/11	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
11/00	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
11/01	.5	.5	.5	.5	1	**1**	1	1	0	0	0	0	.5	**.5**	.5	.5
11/10	.5	.5	.5	.5	0	0	0	0	1	1	**1**	1	.5	.5	**.5**	.5
11/11	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5

Table 1: payoff values between the 16 CS relative to simulation 1

Simulation 2

In the second simulation we will still consider a non-spatially organized population. The only difference in respect to the previous simulation is that now we reward both the speakers and the listeners. Figure 2 shows a snapshot of the new simulation keeping $\mu = 0.02$. We notice that the red optimal communication system 10/10 emerges almost immediately and it is able to maintain the domain of the entire population.

Figure 3 shows how the percentage of the dominant communication system (after 1000 iterations) changes with the change of μ. The graph reports the average percentage and fitness out of 10 runs of the simulation. When $\mu = 0$ there is some run where no full dominance is obtained (see Special Case 1). From the plot it is clear that the percentage of the dominant communication system decreases linearly with the increase of the mutation rate: for $0 < \mu \lesssim 0.10$ an optimal communication system can invade and keep domain of the entire population (staying above chance frequency). With higher mutation rate no communication system can dominate the population and the fitness stays at chance level.

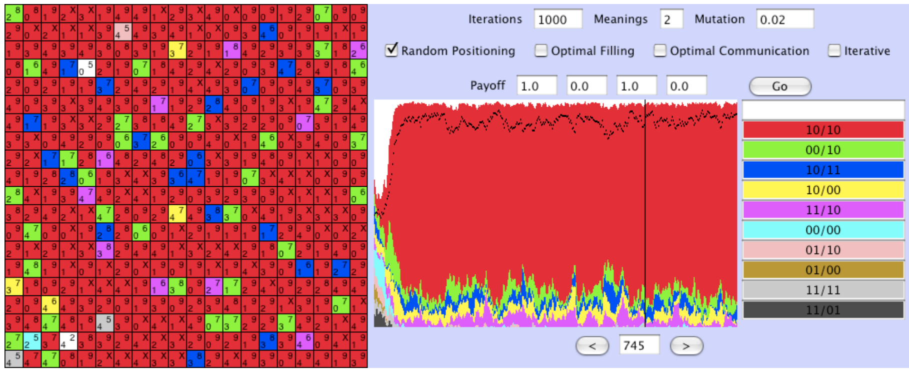

Figure 2: snapshot simulation 2

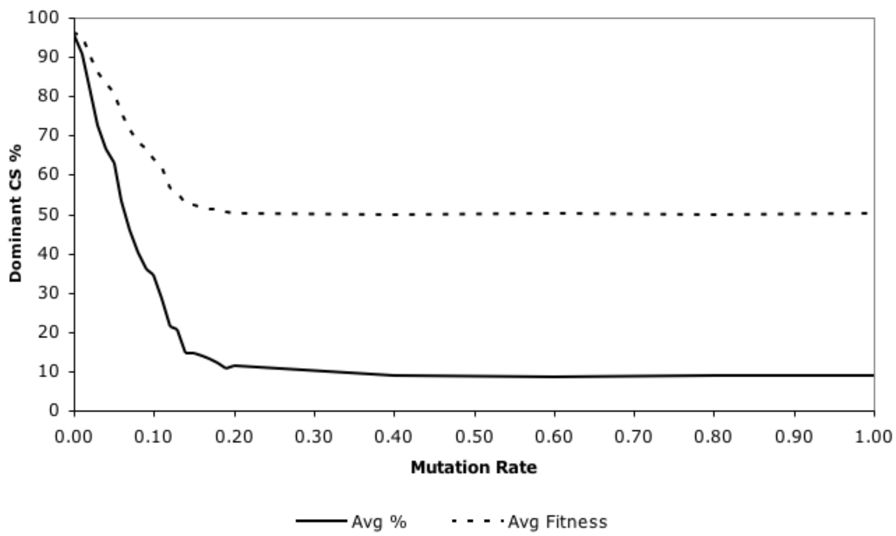

Figure 3: graph simulation 2

Simulation 3

In the third case we will start again from the default settings of Simulation 1 (only listeners are rewarded) with an iterative communication mode. We want to see if the capability of keeping a primary communication system in "trusted neighborhoods" can in itself allow the emergence of a Saussurean Communication within the population.

The snapshot of Figure 4 reports one possible simulation run. In the iterative mode each agent in the left side panel is split in two parts: the bottom left one is colored with the primary communication system and the upper right with the

secondary one.
The optimal red communication system 10/10 is able to invade after around 200 iterations and keeps the domain of the entire population until the last iteration. It is important to note from the snapshot that, once the Saussurean Communication takes place, the majority of the secondary transmission systems differs from the red primary transmission system. If for some chance it would drift to the primary transmission system, any non trusted opponent would benefit from it. This possibility mentioned in Oliphant's paper as a major source of instability seems to affect very few simulations and mainly in cases where the mutation rate is kept very low ($\mu \lesssim 0.005$).

Figure 4: snapshot simulation 3

The graph in Figure 5 shows the percentage of the dominant communication system with increasing μ. Here there is a relatively sudden jump around $\mu = 0.05$. After this threshold the dominant communication system stays at chance frequency. It is also possible to see a rise in fitness from $0 < \mu < 0.01$ This confirms our previous result that for very low mutation rates, a random drift leading to the optimality of the secondary transmission system, can compromise the maintenance of a Saussurean Communication.
The results seem to prove that the iterative communication mode can be a sufficient element to allow the emerge and maintenance of an optimal communication system for $0.01 < \mu < 0.05$. The analogue iterated mode simulation reported by Oliphant, characterized by an instability of the Saussurean Communication, considers the case in which the mutation rate falls in the critical region (~ 0.004[1]).

In the actual implementation of the iterative mode, if the speaker fails/succeeds to convey a certain meaning, only the listener stops/starts to trust the oppo-

[1]The mutation system introduced in Oliphant is based on mutation affecting the entire genome (.27%) and crossing-over. The existence of a precise way to convert it into our metric is in doubt.

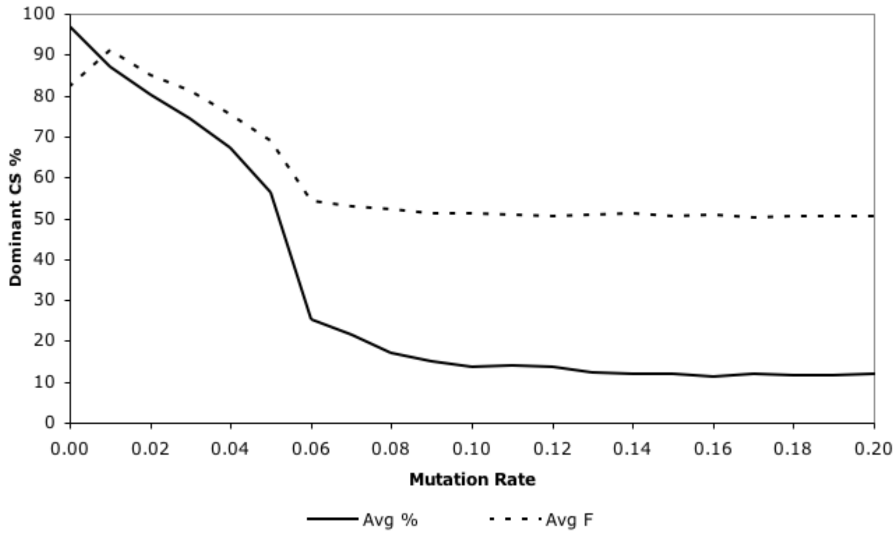

Figure 5: graph simulation 3

nent[2]. We believe that this is how Oliphant's simulation works. We observe a remarkable difference in the outcome if both the listener and the speaker are affected by the failure or success of the last communication attempt. In this new circumstance the secondary communication system is more susceptible to random drift allowing a cyclic shift between the two optimal communication systems as shown in Figure 6. Figure 7 plots the percentage of the dominant population of this special case. If compared with the dynamics in Figure 5 we can consider some qualitative differences: in the new setting the initial rising in fitness is slightly shifted to the right and the dominant population percentage reaches chance level much more sharply.

Simulation 4

So far we have been dealing with non spatially organized population: the position of each has been randomly shuffled at each iteration. This perfectly represents the case in which no spatial organization is taken into account.

In the last simulation we want to see how the outcome of the simulation 1 is affected when the positions of the agents are preserved through their life cycles. This basically means that each agent keeps the same exact neighbours throughout its 5 life stages.
Figure 8 shows how spatial organization is sufficient for Saussurean Commu-

[2]This is particular true for this setting where the listener is the only one affected by the success or failure of the communication.

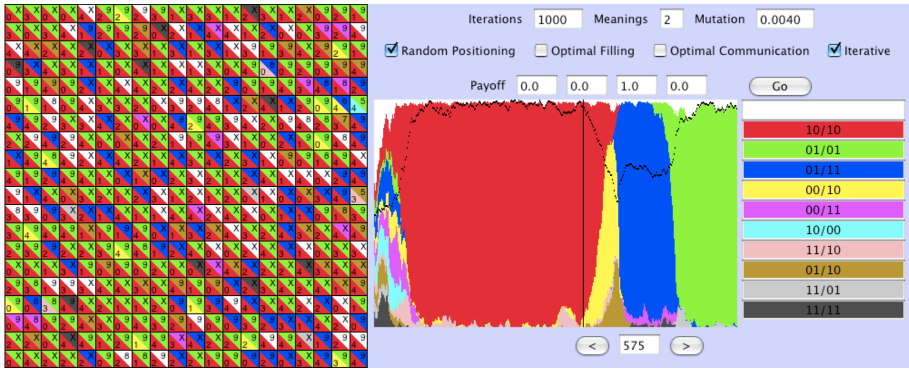

Figure 6: snapshot simulation 3 with modified iterative mode

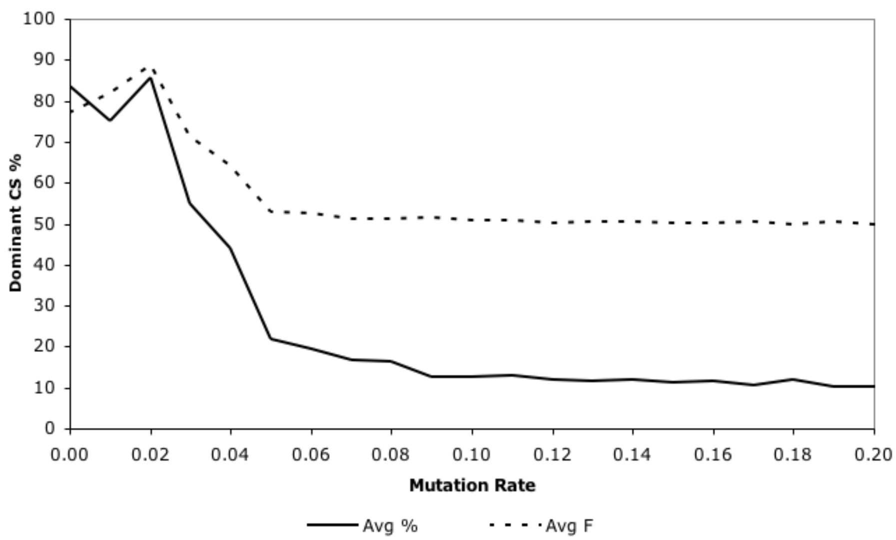

Figure 7: graph simulation 3 with modified iterative mode

nication to emerge. It is evident how "small communities" sharing a suboptimal communication systems are able to survive above chance frequency. This "parish effect" phenomenon applies only to communication systems compatible with the dominant one. In the snapshot the green, blue, and violet populations are example of surviving parishes which are able to understand the dominant red community, without having high payoff when communicating among themselves.

As in all the previous simulations Figure 9 plots the changing of the percentage of the dominant communication system with the changing of the mutation rate μ. The dominant population decreases sharply when the mutation rate is close

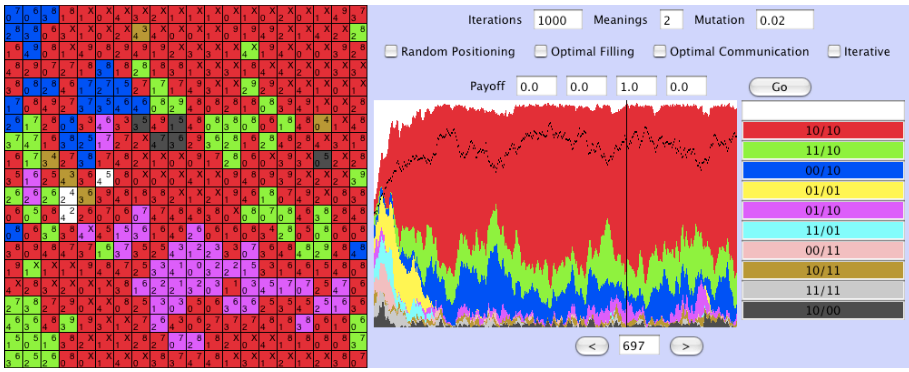

Figure 8: snapshot simulation 4

to zero and slowly reaches change frequency around $\mu = 0.2$ with a characteristic hyperbolic shape. This seems to suggest that under these parameters the percentage of the dominant communication system is inversely proportional to the mutation rate.

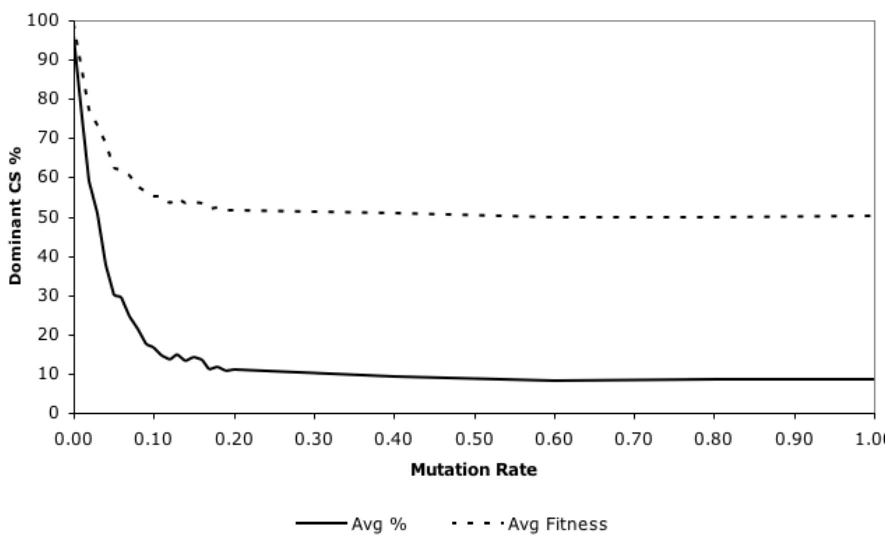

Figure 9: graph simulation 4

Case studies

In this section we will take into consideration 3 case studies which will be analyzed in detail within a game theoretical framework. As before we will keep $N = 2$.

Case Study 1 - Two symmetric communication systems

We will consider here the case where both speakers and listeners are rewarded in a non spatially organized population (simulation 2) with $\mu = 0$.
The most probable outcome of this setting is the absolute dominance of an optimal communication system. A rather rare exception is reported in Figure 10. This simulation can be divided in two main phases. During the first part (until iteration 964) 3 main communication systems are present in the population: red (11/01), green (01/11) and blue (01/00). The red communication system is characterized by an optimal transmitting system and a suboptimal receiving system while the green and the blue have a suboptimal transmitting system and an optimal receiving system. It's important to notice that the green and the blue have the same payoff when communicating with the same opponent. For this reason we can conclude that the population is in a equilibrium state: half is red and the other half is green and blue. During the second phase the blue population is completely replaced by the green population. Since no selective pressure has caused this outcome we say that the blue population goes extinct by random drift.

Table 2 reports the specification of the 3 communication systems, the payoff function and the derived population dynamics equation. Figure 11 shows the 3D geometric interpretation of this equation: the plane represts the equation $x_R + x_G + x_B = 1$ which is the space of all possible frequency combinations of the 3 communication systems. While there is no horizontal selective pressure (between green and blue) we have a vertical point of attraction in $x_R = 1/2$. This bias of attraction is also evident in the 2D illustration which is a vertical section of the plane.

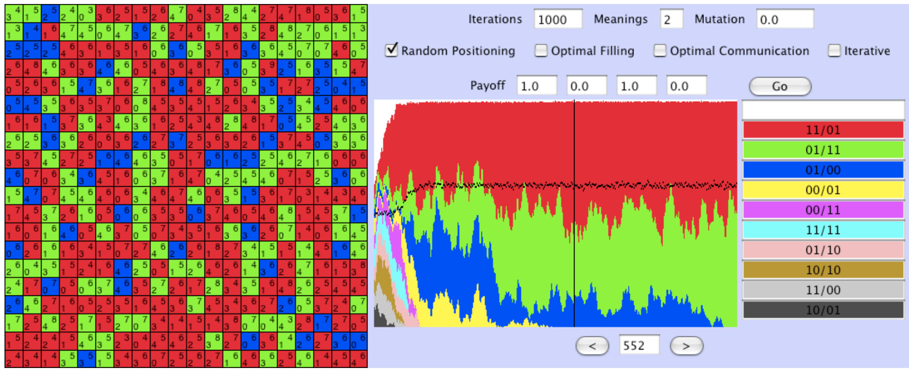

Figure 10: snapshot of case study 1

$R = 11/01$

T_R	f_0	f_1
m_0	0	1
m_1	0	1

R_R	f_0	f_1
m_0	1	0
m_1	0	1

$G = 01/11$

T_G	f_0	f_1
m_0	1	0
m_1	0	1

R_G	f_0	f_1
m_0	0	0
m_1	1	1

$B = 01/00$

T_B	f_0	f_1
m_0	1	0
m_1	0	1

R_B	f_0	f_1
m_0	1	1
m_1	0	0

F	R	G	B
R	1/2	3/4	3/4
G	3/4	1/2	1/2
B	3/4	1/2	1/2

$$\begin{cases} f_R = x_R \cdot F_R(R,R) + x_G \cdot F_R(R,G) + x_B \cdot F_R(R,B) = \frac{1}{2}x_R + \frac{3}{4}x_G + \frac{3}{4}x_B \\ f_G = x_R \cdot F_G(R,G) + x_G \cdot F_G(G,G) + x_B \cdot F_G(G,B) = \frac{3}{4}x_R + \frac{1}{2}x_G + \frac{1}{2}x_B \\ f_B = x_R \cdot F_B(R,B) + x_G \cdot F_B(G,B) + x_B \cdot F_B(B,B) = \frac{3}{4}x_R + \frac{1}{2}x_G + \frac{1}{2}x_B \end{cases}$$

Table 2: case study 1 a) red, green and blue transmitting and receiving systems b) payoff function c) population dynamics equation

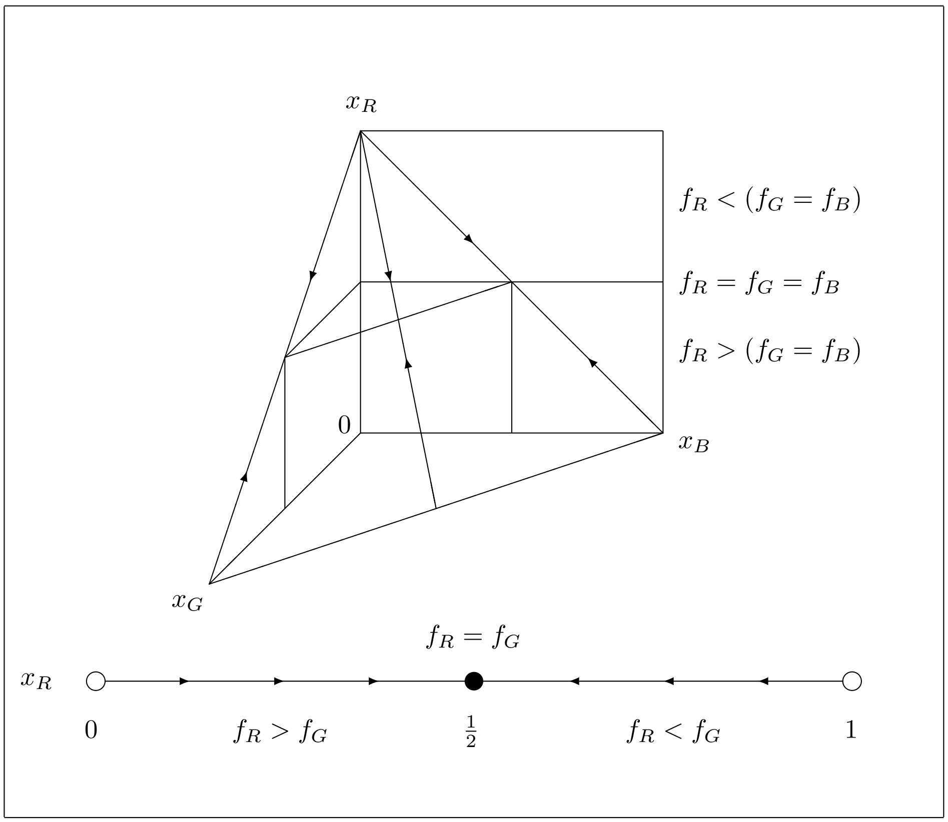

Figure 11: graphical representation of population dynamics of case study 1

Case Study 2 - Two asymmetric communication systems

In the second case study we will consider an other exceptional case of Simulation 2 with $\mu = 0$. In this outcome (Figure 12) the red population has an optimal transmitting system and suboptimal receiving system while the green has an optimal speaking an an optimal reception system in respect to the red, and opposite in respect to itself. The green population will therefore have a zero payoff when its members communicate among themselves. This lead to a stable situation where this population is 1/4 of the entire population. Table 3 reports the details of two communication systems and the population dynamics.

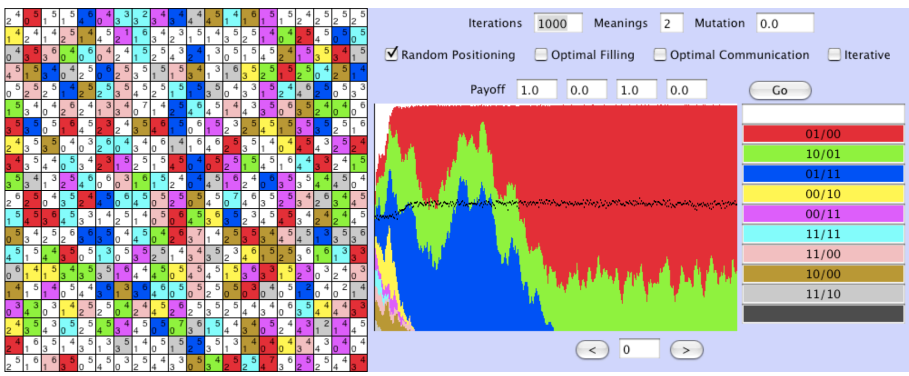

Figure 12: snapshot of case study 2

$R = 01/00$

T_R	f_0	f_1
m_0	1	0
m_1	0	1

R_R	f_0	f_1
m_0	1	1
m_1	0	0

$G = 10/01$

T_G	f_0	f_1
m_0	0	1
m_1	1	0

R_G	f_0	f_1
m_0	1	0
m_1	0	1

F	R	G
R	1/2	3/4
G	3/4	0

$$\begin{cases} f_R = x_R \cdot F_R(R,R) + x_G \cdot F_A(R,G) = \frac{1}{2}x_R + \frac{3}{4}x_G \\ f_G = x_R \cdot F_G(R,G) + x_G \cdot F_G(G,G) = \frac{3}{4}x_G \end{cases}$$

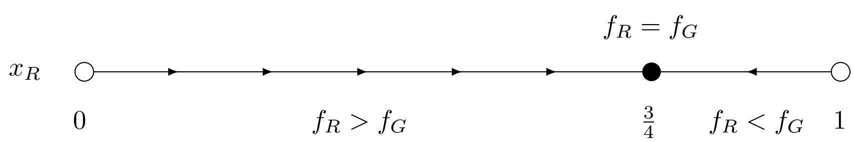

Table 3: case study 2 a) red, green transmitting and receiving systems b) payoff function c) population dynamics equation d) graphical representation of population dynamics

Case Study 3 - Two symmetric communication systems with only listeners rewarded

In the last case study we consider the case where only listeners are rewarded in a non spatially organized population (simulation 1) with $\mu = 0$. In this case we have an equilibrium between the red and the green population each occupying half of the space. The red population has a high payoff when communicating with the green but zero payoff against itself. Table 4 reports the specification of the population dynamics.

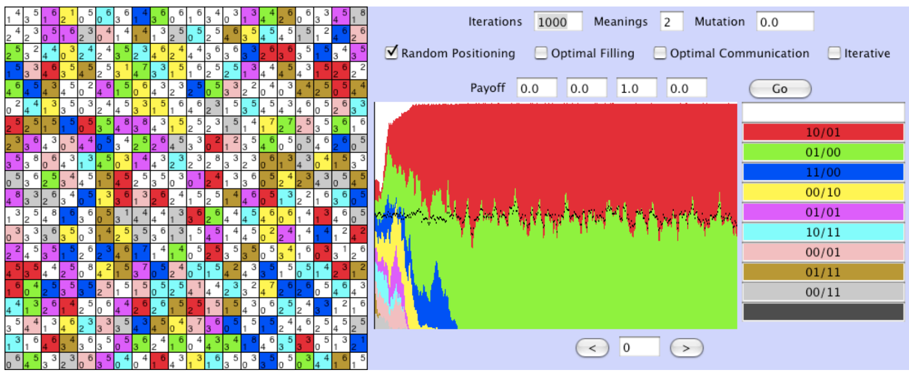

Figure 13: snapshot of case study 3

$R = 10/01$

T_R	f_0	f_1
m_0	0	1
m_1	1	0

R_R	f_0	f_1
m_0	1	0
m_1	0	1

$G = 01/00$

T_G	f_0	f_1
m_0	1	0
m_1	0	1

R_G	f_0	f_1
m_0	1	1
m_1	0	0

F	R	G
R	0	1
G	1/2	1/2

$$\begin{cases} f_R = x_R \cdot F_R(R,R) + x_G \cdot F_R(R,G) = x_G \\ f_G = x_R \cdot F_G(R,G) + x_G \cdot F_G(G,G) = \frac{1}{2}x_R + \frac{1}{2}x_G \end{cases}$$

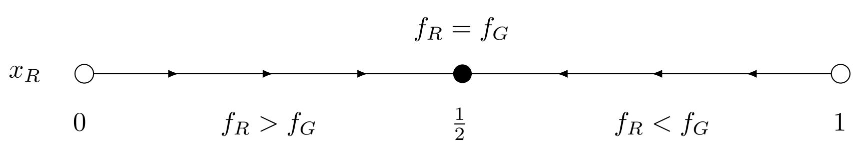

Table 4: case study 3 a) red, green transmitting and receiving systems b) payoff function c) population dynamics equation d) graphical representation of population dynamics

Analytic analysis of communication systems

Introduction

This section will present a mathematical framework for the analysis of the evolution and maintenance of communication conventions, according to the mathematical model used in Nowak et al. [2]. We will compare this analysis with the empirical results we can obtain from our simulation. We will limit our study to the case where both speakers and listeners are rewarded in a non spatially distributed population (as in Simulation 2).

The two models

In order to compare the two models we will need to define our set of variables:

$\mathbf{N}$	is the size of the meanings space (as before).
$\mathbf{G}$	is the number of communication systems or grammars.
$\mathbf{d_{ij}}$	is the number of mappings differing between grammar i and j.
$\mathbf{e_{ij}}$	is the number of mappings in common between grammar i and j.
$\mathbf{A}$	is the compatibility matrix specifying to which degree grammar i is compatible with grammar j.
$\mathbf{F}$	is the payoff matrix specifying the payoff between any two grammars i, j.
$\mathbf{x_i}$	is the frequency of individuals using grammar i.
$\mathbf{f_i}$	is the average payoff of a individuals using grammar i.
$\mathbf{Q}$	is the acquisition matrix specifying for each i, j what is the probability that an agent learning from a teacher with grammar i will end up speaking grammar j.
ϕ	is the average fitness of the population.
$\dot{\boldsymbol{x}}_{\mathbf{i}}$	is the population dynamic of grammar i: it represents how much the frequency of grammar i changed from last iteration (in a continuous timeline it is the derivate of frequency with respect to time).

The main difference between the two models lies on the complexity behind the grammars of the agents. In Nowak's model the grammars are assumed to be equally distant from one another while in Oliphant's the hierarchical structure behind the communication system leads to a more complex distance space of the type found in Table 1. This difference causes the mutation rate to have different scopes in the two models: in Nowak's μ is related to the probability of going from one grammar to any other grammar, while in Oliphant's μ refers to the probability of changing each single unit of the grammar. In Table 5 we report how the formulas behind the two models are related.

Results

Oliphant's model results in a more complex population dynamics formula which is hard to derive analytically. We therefore choose to approximate the result numerically. In the first iteration the frequency of each grammar is set: in an

	Nowak	Oliphant
$\mathbf{A(i,j)}$	1 if $i = j$ 0.5 if $i \neq j$	$\frac{\sum_{\mathbf{i}} \mathbf{R_j(T_i)=i}}{\mathbf{N}}$
$\mathbf{F(i,j)}$	$\frac{A(i,j)+A(j,i)}{2}$	$\frac{A(i,j)+A(j,i)}{2}$
$\mathbf{f_i}$	$\sum_j x_j F(i,j)$	
$\mathbf{Q(i,j)}$	$1-\mu$ if $i = j$ $\frac{\mu}{G-1}$ if $i \neq j$	$(\frac{\mu}{N})^{d_{ij}} \cdot (\frac{\mu}{N} + (1-\mu))^{e_{ij}}$
ϕ	$\sum_k x_k f_k$	
$\dot{x}_i$	$(\sum_{j=1}^{n} x_j f_j Q(j,i)) - \phi x_i$	

Table 5: Math behind the population dynamics of Nowak and Oliphant

Optimal Start the entire population shares the same optimal grammar while in a *Random Start* all the grammars have equal initial frequency.
In each iteration the new frequency of each grammar is calculated according to the population dynamic formula of Table 5. A population is consider to have reached a linguistic equilibrium when the new iteration is equal to the previous one (up to a tolerance of 10^{-5}).

We start by showing the result of the numerical approximation of Nowak's model. In Figure 14 we can see that the approximation leads to the *Error Threshold Effect* found Nowak: in a population initialized with an "Optimal Start", there exist a specific value[3] of acquisition error (μ) above which the optimal communication is not able to be maintained in the population.

If we now turn the attention to our simulation we would like to find out whether we can reproduce any similar phenomenon. In Figure 3 we saw already how the dominant communication system changes with the increase of μ in a "Random start" mode, and we couldn't find any threshold behaviour there. Figure 15 shows the same dynamics together with the "Optimal Start" counterpart. This doesn't seem to add any advantage: the dominant communication system linearly drops with the increase of μ from the very first beginning whether the initialization is optimal or random. In Figure 16 we present the same result we can derive numerically from the formula of Oliphant section of Table 5.

In the simulations we run so far, apart from the "iterative mode", we have encountered a unique communication scheme: one single meaning is picked at random in each communication between two agents. We now introduce a new constraint in the communication system, the **All or Nothing** (AoN) mode, where all the meanings are used in the communication between two agents. Furthermore the payoff of the communication is 1 if and only if all the meanings are correctly communicated (0 otherwise).

[3]We notice that this specific value seems to be somehow shifted compared to the one reported in Nowak. In our case the threshold occurs on $\mu = 0.16$ while in Nowak $\mu = 0.07$.

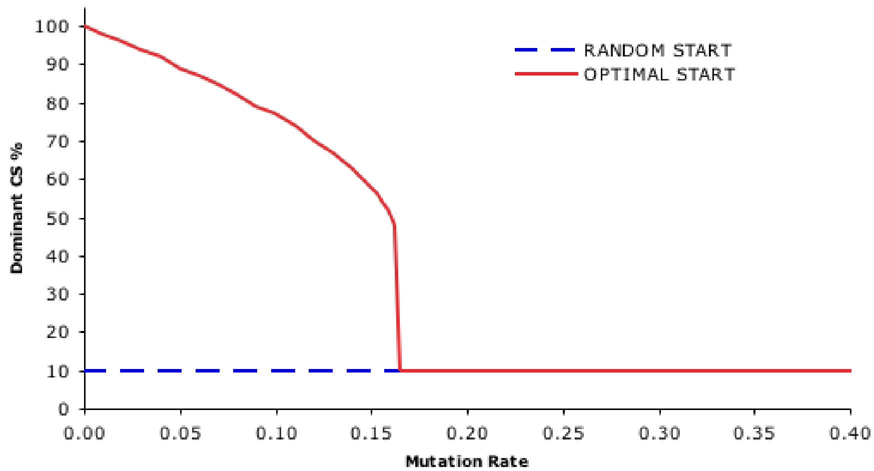

Figure 14: Numeric approximation of Nowak population dynamics with a_{ij}=0.5 and $G = 10$

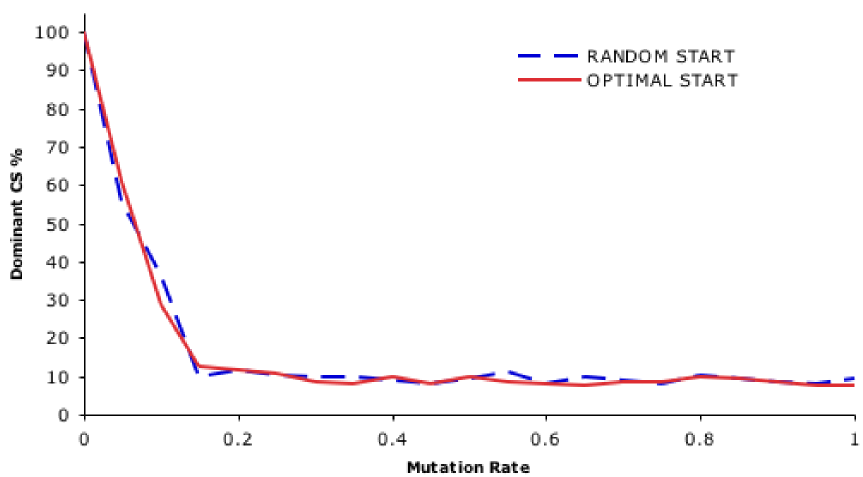

Figure 15: Simulation in standard mode with N=2 (G=16)

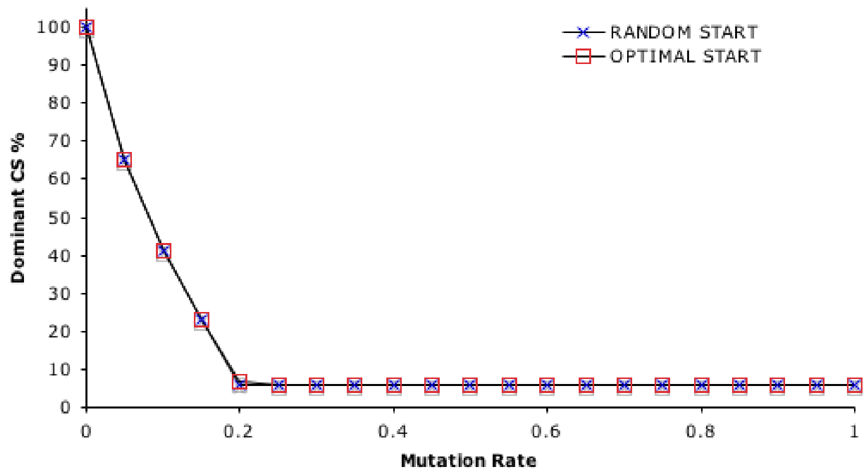

Figure 16: Numeric approximation of Oliphant population dynamics in "Normal" mode with N=2 (G=16)

We want to test whether AoN can bring us closer to the threshold effect. Figure 17 shows the dominant communication system dynamics from our simulation in the new mode with N=2. The frequency is now decreasing in a qualitative different way than before but still far away from replicating the threshold dynamics. Figure 18 shows a similar dynamics for N=10.

We try to replicate such dynamic using numeric approximation. We first modify the formula behind the payoff F_{ij} between any two grammars i and j (Table 5). While before it was defined as the average of their reciprocal compatibilities, we now use the floor value of the average (F_{ij} is rounded to 0 whenever the average of the reciprocal compatibilities is less than 1). Figure 19 shows the new dynamic with $N = 2$. Although the curve looks pretty similar to the one we encountered before in Figure 17, it doesn't seem to have the expected hyperbolic shape. If we now analyze the F matrix, we realize that the constraint we imposed, selects 4 communication system pairs to have positive (and maximum) fitness. Besides the trivial case of the 2 optimal grammars (10/10, 01/01) each having positive fitness when communicating with itself, we have 2 other complementar grammars (01/10, 10/01) wich score positive fitness when communicating with one another. In order to exclude the latter pair, we have to put a new constraint, restricting a pair of grammars to have maximum payoff only when each separately has maximum fitness against itself. Figure 20 shows the expected dynamic with this last modification.[4]

[4]Unfortunately our numeric approximation cannot verify our simulation result in the case where N=10. In order to do so we would have to deal with 10^{20} possible grammars.

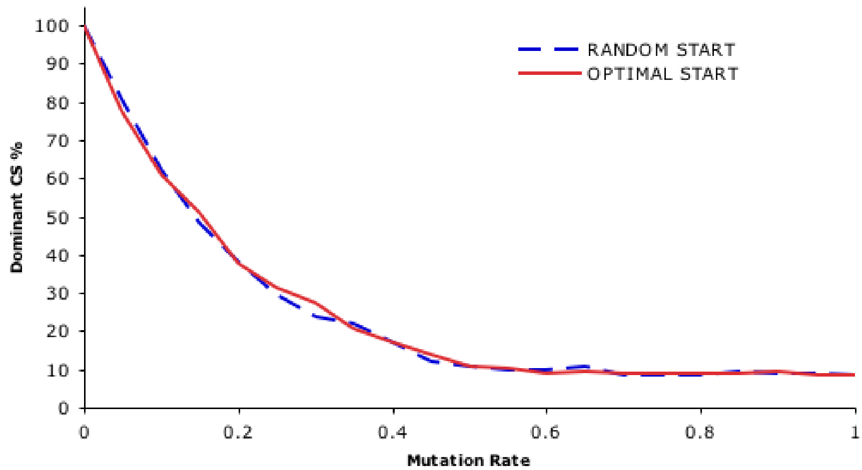

Figure 17: Simulation in "All or Nothing" mode with N=2 (G=16)

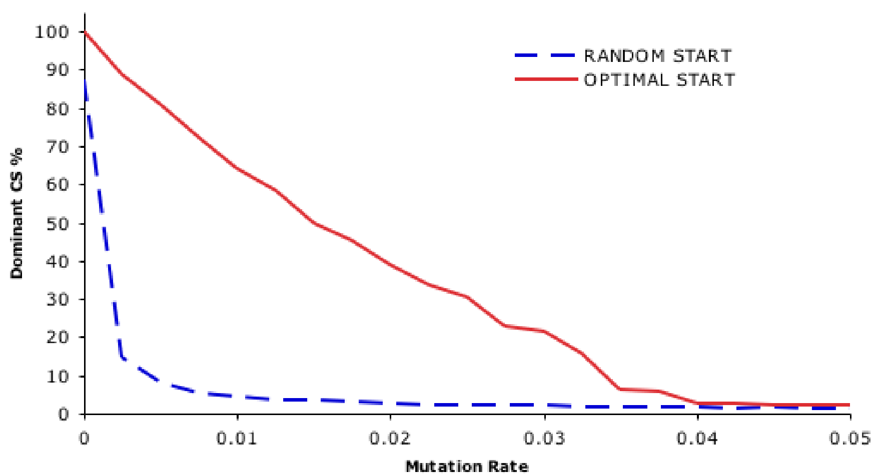

Figure 18: Simulation in "All or Nothing" mode with N=10 (G=10^{20})

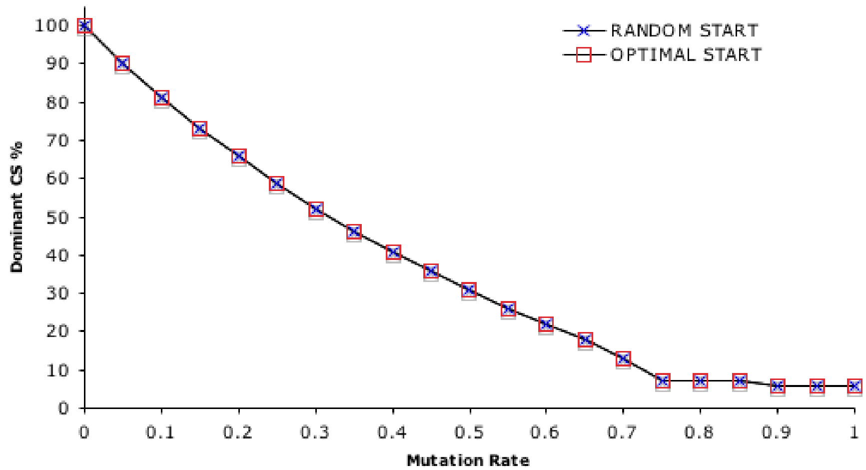

Figure 19: Numeric approximation of Oliphant population dynamics in “All or Nothing” mode (first version) with N=2 (G=16)

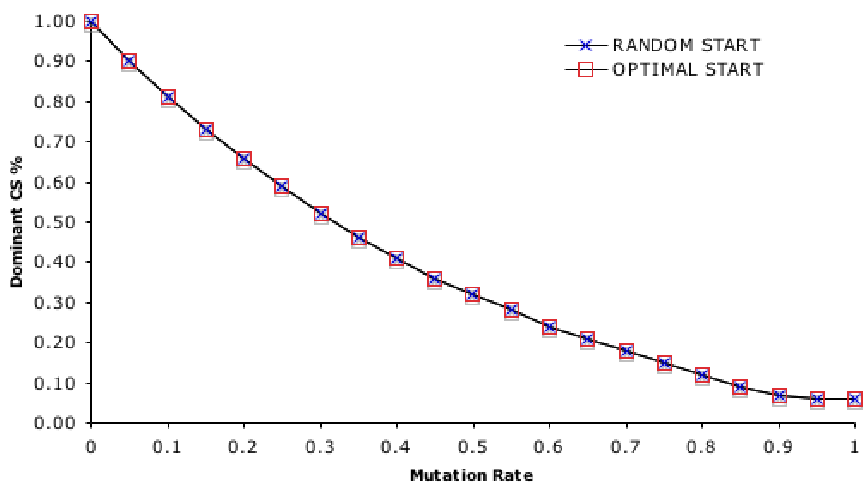

Figure 20: Numeric approximation of Oliphant population dynamics in “All or Nothing” mode (second version) with N=2 (G=16)

We would like to conclude this section with a further constraint which we can apply to our simulation, in order to replicate the threshold effect: the **Absolute All or Nothing** mode (AAoN). In this new setting all the meanings are used in the communication between two agents as in AoN. The payoff of an agent is 1 if and only if all the meanings are correctly communicated in all the communications (0 otherwise). Figure 21 shows this result. So far, we found no way to obtain the same result with numeric approximation.

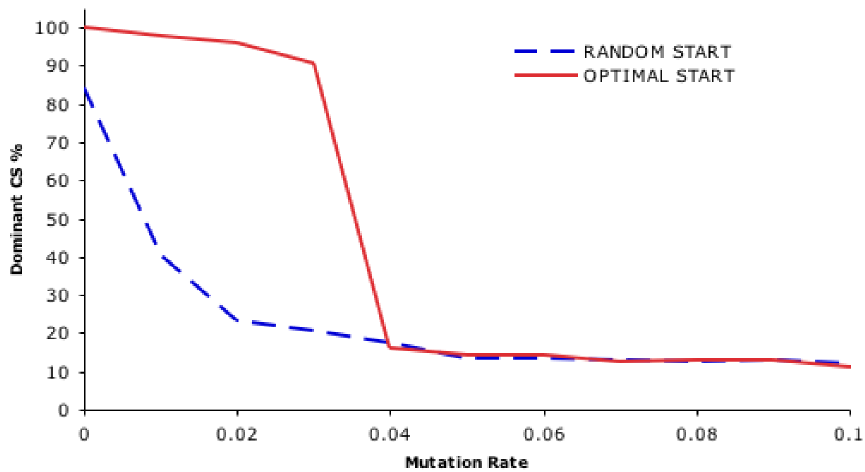

Figure 21: Simulation in "Absolute All or Nothing" mode with N=2 (G=16)

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[2] Martin A. Nowak, Natalia Komarova, and Partha Niyogi. Evolution of universal grammar. *Science*, 291:114–118, 2001.

[3] Michael Oliphant. The dilemma of Saussurean communication. *BioSystems*, 37(1-2):31–38, 1994.