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Dual Currency Circulation and Information

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Abstract

This paper studies dual money circulation within a model *à la* Lagos and Wright [18]. The main result of the analysis is the following. In a dual currency circulation economy, (i) the money type with an higher rate of growth circulates more widely than the money type with a lower rate of growth, and (ii) the two monies are symmetrically accepted if and only if they grow at the same rate.

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> "A long-held notion is that an inferior currency should circulate more widely than a superior money. Those holding both monies would prefer spending the 'bad' money as soon as they can, and keep the 'good' money for future purchases," [4].

1 Introduction

In many transitional economies, citizens may adopt a dual-payment system by using the foreign currency – e.g. the dollar or the euro – in addition to their own locally issued currency as a mean of exchange. In these countries it is usually observed that the foreign currency is not universally accepted, however, in the sense that the great majority of agents make use of the local currency in transactions. This phenomenon rises the following questions. First, under what conditions do the foreign and local currencies co-exist? Second, when is the local currency more widely accepted than the foreign currency and why? Third, what is the role of monetary policy in each case? The focus of this paper is to address these questions within a third-generation search-theoretic model of money.

Search-theoretic models of monetary economics have become the dominant framework to study dual currency circulation as they formalize the essential role of money explicitly, rather than assuming it exogenously. The search literature on multiple currencies includes, to cite a few, [1], [7], [8], [12], [21], [23], and [25]. All these papers share the assumption that money is indivisible, and individual holdings of money are bounded at one unit (first- and second-generation models). An exhaustive survey of these models is in [5]. Some attempts to study multiple-money holdings in a two-currency framework are [4], [6], and [10]. In [4], agents are allowed to hold two units of indivisible money. [10] relax the restriction of money indivisibility, but they assume that goods are indivisible. In their paper, [6] derive numerical results within a divisible-money divisible-good model.

This paper studies the links between monetary policy and dual currency circulation by extending Lagos and Wright [18] (hereafter, LW) to two currencies and imperfect information. The LW framework is useful because allows one to derive analytical results while keeping the distribution of money holdings tractable (third-generation models). Imperfect information opens the door to restrictions on which medium of exchange must be used for payment in goods trade. This is because, for instance, money types can be counterfeit and not all agents can recognize counterfeits from authentic monies, so they will never accept some money types. Other works assume that financial assets ([14], [16, 17] and [19]) and capital goods ([15]) can be used with money as competing media of exchange.

There are a number of papers that are closely related to the present work (i.e. [9], [11], and [20]). They also study two-currency issues within a third-generation model of money, but never consider the role of information.

The main result of the paper is the following. In a dual currency circulation economy, (i) the local money circulates more widely than the foreign money, and (ii) local and foreign monies are symmetrically accepted if and only if they grow at the same rate.

The paper is organized as follows. Section 2 describes the basic framework and the agents' decision problem. Stationary equilibria are characterized in Section 3. Section 4 introduces imperfect information. Section 5 derives conditions for endogenous acceptability. The conclusions end the paper.

2 The model

The basic setup is borrowed from LW. Time is indexed by $t \in \{1, 2, ..., \infty\}$ and each period t is divided into two subperiods where different activities take place. There is a $[0, 1]$ *continuum* of infinitely-lived agents and two types of perfectly divisible commodities – general and special goods. Each agent produces a subset and consumes a different subset of the special goods. Specialization is modeled as follows. In the first subperiod, each agent meets someone who produces a good he wishes to consume with probability $\sigma \in \mathbb{R}\,(0, 1/2]$ and meets someone who likes the good he produces with the same probability σ. With probability $1 - 2\sigma$ an agent has no opportunity to trade. Let us denote consumers as buyers and producers as sellers. The specialization of agents over consumption and production of the special good gives rise to a 'double coincidence of wants' problem. In contrast to special goods, general goods can be consumed and produced by all agents.

Special goods can only be produced during the first subperiod, while general commodities can only be produced during the second subperiod. In the first subperiod, agents participate in a decentralized market (DM) where each meeting is bilateral and is a random draw from the set of pairwise meetings. In this market the terms of trade are determined by bargaining. In the second subperiod agents produce general goods and can trade in a centralized market (CM).

Agents get utility $u(q)$ from q consumption in the DM, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $u(0) = 0$. Furthermore, it is assumed that the elasticity of utility $e(q) = qu'(q)/u(q)$ is bounded. Producers incur utility cost $c(q)$ from producing q units of output with $c'(q) > 0$, $c''(q) \geq 0$, and $c(0) = 0$. Let q^* denote the solution to $u'(q^*) = c'(q^*)$.

In the CM all agents consume and produce, getting utility $U(x)$ from x consumption, with $U'(x) > 0$, $U'(0) = \infty$, $U'(\infty) = 0$, $U''(x) \leq 0$ and $U(0) = 0$. Let x^* be the solution to $U'(x^*) = 1$. All agents can produce consumption goods from labor using a linear technology. Agents discount between the CM and the next-period DM, but not between DM and CM. This is not restrictive since as in [22] all that matters is the total discounting between one period and the next. It is assumed that individual actions are not observable in the CM so as to avoid contagion equilibria (see [2, 3]).

All agents are *anonymous* in the DM. Consequently, trade credit is ruled out and transactions are subject to a *quid pro quo* restriction so there is a role for a medium of exchange ([13] and [24]).

At the beginning of a period, the expected steady state lifetime utility of the representative agent is

$$(1-\beta)\mathcal{W} = \sigma\left[u(q) - c(q)\right] + U(x) - x \tag{1}$$

where $\beta \in \mathbb{R}(0,1)$ is the discount factor and q the quantity of goods consumed by a buyer and produced as a seller in the DM. The solution to the planner problem in an economy without anonymity yields

$$U'(x^*) = 1, \tag{2}$$

$$u'(q^*) = c'(q^*). \tag{3}$$

These are the quantities chosen by a social planner who could force agents to produce and consume.

Figure 1: Timing of events

There are two types $j \in \{1, 2\}$ of durable and intrinsically useless objects: money 1 and money 2. It is assumed that two independent central banks exist that control the supply of each money type at any time t, $M_{j,t} > 0$. It is also assumed that $M_{j,t} = \gamma_j M_{j,t-1}$, where $\gamma_j > 0$ is constant and new money of type j is injected, or withdrawn if $\gamma_j < 1$, as lump-sum transfers $\pi_j M_{j,t-1} = (\gamma_j - 1) M_{j,t-1}$ to all agents. The analysis will be restricted to policies where $\gamma_j \geq \beta$. The time subscript t is omitted and shorten $t + 1$ to $+1$, etc. in what follows. Let us introduce the following:

Notation 1 *If money 1 grows more quickly than money 2 (i.e. $\gamma_1 > \gamma_2$), then money 1 is referred to as the 'local' money and money 2 as 'foreign' money, and vice versa.*

The timing of events is shown in Figure 1. At the beginning of the DM, bilateral trade of goods begins. In the CM agents receive lump sum transfers, produce, consume and rebalance their money holdings. The structure of this economy is shown in Figure 2.

At time t, let $\phi_j = 1/P_j$ be the real price of money j and P_j the price of goods in term of money j in the CM. The focus is on symmetric steady state equilibria, where aggregate real money balances are constant over time, i.e.

$$\phi_j M_j = \phi_{j,-1} M_{j,-1} \tag{4}$$

$\forall j \in \{1, 2\}$, which implies that $\phi_{j,-1}/\phi_j = M_j/M_{j,-1} = \gamma_j$; the Fisher equation holds, hence it is equivalent to set the nominal interest or inflation.

Figure 2: Environment

3 Stationary equilibria

Consider a symmetric stationary equilibrium. Let $V(\mathbf{m})$ denote the expected value from trading in the DM with a portfolio $\mathbf{m} \equiv (m_1, m_2)$ of monies. Let $W(\mathbf{m})$ denote the expected value from entering the CM with a portfolio $\mathbf{m}$ of monies.

In the CM agents produce h units of good using h hours of labor, receive lump sum transfers, consume, and adjust their money balances. The real wage per hour is normalized to one. Hence, the representative agent's problem is

$$W(\mathbf{m}) = \max_{x,h,\mathbf{m}_{+1}} \left[U(x) - h + \beta V_{+1}(\mathbf{m}_{+1})\right] \tag{5}$$

such that

$$x + \boldsymbol{\phi}\mathbf{m}_{+1} = h + \boldsymbol{\phi}\mathbf{m} + \boldsymbol{\phi}\mathbf{T} \tag{6}$$

where the vector $\mathbf{m}_{+1} \equiv (m_{1,+1}, m_{2,+1})$ is the portfolio of monies taken into period $t{+}1$, $\boldsymbol{\phi} \equiv (\phi_1, \phi_2)$ the real price of monies, and $\mathbf{T} \equiv (\pi_1 M_{1,-1}, \pi_2 M_{2,-1})$ the lump sum transfers. Eliminate h from (5) using (6) and get

$$\begin{aligned} W(\mathbf{m}) = {} & \boldsymbol{\phi}\left[\mathbf{m} + \mathbf{T}\right] \\ & + \max_{x,\mathbf{m}_{+1}} \left[U(x) - x - \boldsymbol{\phi}\mathbf{m}_{+1} + \beta V_{+1}(\mathbf{m}_{+1})\right]. \end{aligned} \tag{7}$$

The first order conditions (FOCs) with respect to x and $\mathbf{m}_{+1}$ are

$$\begin{aligned} U'(x) &= 1, \\ \beta V_{j,+1}(\mathbf{m}_{+1}) &= \phi_j \end{aligned} \tag{8}$$

$\forall j \in \{1,2\}$, where the term $\beta V_{j,+1}(\mathbf{m}_{+1})$ is the marginal benefit of taking money j out of the CM and ϕ_j is its marginal cost.

Two comments are in order here. First, the quantity of goods x consumed by every agent is equal to the efficient level x^*, where $U'(x^*) = 1$. Second, $\mathbf{m}_{+1}$ is independent of $\mathbf{m}$. As a result, the portfolio of monies is degenerate at the beginning of the following period. This is due to the quasi-linearity assumption in (5), which eliminates the wealth effects on money demand in the CM. Agents who bring too much cash into the CM spend some buying goods, while those with too little cash sell goods.

The envelope conditions are

$$W_j(\mathbf{m}) = \phi_j \tag{9}$$

$\forall j \in \{1,2\}$, where $W_j(\mathbf{m})$ is the derivative of $W(\mathbf{m})$ with respect to m_j.

An agent who has a portfolio $\mathbf{m}$ of monies at the opening of the DM has expected lifetime utility

$$\begin{aligned} V(\mathbf{m}) = {} & \sigma\left[u(q) + W(\mathbf{m} - \mathbf{z}_b)\right] \\ & + \sigma\left[-c(q) + W(\mathbf{m} + \mathbf{z}_s)\right] \\ & + (1-2\sigma) W(\mathbf{m}) \end{aligned}$$

where $\mathbf{z}_b \equiv (z_{1,b}, z_{2,b})$ is the amount of monies given up when a buyer and $\mathbf{z}_s \equiv (z_{1,s}, z_{2,s})$ the amount of monies received as a seller. From linearity of $W(\mathbf{m})$, expression (7) can be rewritten as

$$W(\mathbf{m}) \equiv W(0) + \boldsymbol{\phi}\mathbf{m}$$

which can be used to rewrite the indirect utility function as follows

$$V(\mathbf{m}) = W(\mathbf{m}) + \sigma\left[u(q) - \boldsymbol{\phi}\mathbf{z}_b\right] + \sigma\left[-c(q) + \boldsymbol{\phi}\mathbf{z}_s\right]. \tag{10}$$

Again, due to linearity of $W(\mathbf{m})$, the Nash bargaining problem in the DM reduces to

$$\max_{q,\mathbf{z}} \left[u\left(q\right) - \boldsymbol{\phi}\mathbf{z}\right]^{\theta} \left[-c\left(q\right) + \boldsymbol{\phi}\mathbf{z}\right]^{1-\theta}$$

such that

$$\mathbf{z} \leq \mathbf{m} \tag{11}$$

where $\theta \in \mathbb{R}\left(0,1\right]$ is the buyer's bargaining power, and $\mathbf{z}_b = \mathbf{z}_s = \mathbf{z}$ the portfolio monies exchanged in a bilateral meeting. The constraint (11) means that buyers cannot spend more monies $\mathbf{z}$ than what they bring into the DM $\mathbf{m}$. The solution to the bargaining problem is

$$\boldsymbol{\phi}\mathbf{z} = g\left(q\right) \equiv \frac{\theta u'(q)c(q)+(1-\theta)u(q)c'(q)}{\theta u'(q)+(1-\theta)c'(q)} \tag{12}$$

where $g\left(q\right) > 0$. Assuming that buyers have all the bargaining power, i.e. $\theta = 1$, equation (12) reduces to $\boldsymbol{\phi}\mathbf{z} = g\left(q\right) = c\left(q\right)$. If $\boldsymbol{\phi}\mathbf{m} \geq c\left(q^*\right)$ then the buyer exchanges $\mathbf{z} < \mathbf{m}$ of his portfolio for the first best quantity q^*. Otherwise, he gives the seller all of his portfolio, $\mathbf{z} = \mathbf{m}$, in exchange for the quantity q that satisfies $c\left(q\right) = \boldsymbol{\phi}\mathbf{m}$. The outcome is independent of the seller's portfolio of monies, and it depends on $\mathbf{m}$ only if the constraint $\mathbf{z} \leq \mathbf{m}$ binds.

Thus, assuming $\theta = 1$, it holds that

$$q\left(\boldsymbol{\phi}\mathbf{m}\right) = \begin{cases} q^* & \text{if } \boldsymbol{\phi}\mathbf{m} \geq c\left(q^*\right) \\ c^{-1}\left(\boldsymbol{\phi}\mathbf{m}\right) & \text{if } \boldsymbol{\phi}\mathbf{m} < c\left(q^*\right) \end{cases} \tag{13}$$

then, using the bargaining solution, the value function (10) can be rewritten as

$$V\left(\mathbf{m}\right) = W\left(\mathbf{m}\right) + \sigma\left[u\left(q\left(\mathbf{m}\right)\right) - c\left(q\left(\mathbf{m}\right)\right)\right]. \tag{14}$$

Now, take the differential of (14) with respect to m_j for $j \in \{1,2\}$ and get

$$V_j\left(\mathbf{m}\right) = W_j\left(\mathbf{m}\right) + \phi_j\sigma\left[\frac{u'(q)}{c'(q)} - 1\right] \tag{15}$$

where $\partial q/\partial m_j = \phi_j/c'\left(q\right)$ has been used. (Notice that if (11) is binding, then $\boldsymbol{\phi}\mathbf{m} = g(q)$.)

By (15), (9), and the second condition in (8) lagged one period, the following holds

$$\frac{\phi_{j,-1}}{\beta} = \phi_j\left\{1 + \sigma\left[\frac{u'(q)}{c'(q)} - 1\right]\right\} \tag{16}$$

$\forall j \in \{1,2\}$. Directly from the Fisher equation, i.e. $1 + i_j = (1+r)(1+\pi_j)$ where $r = 1/\beta - 1$, the following equilibrium condition can be written,

$$\tfrac{\gamma_j - \beta}{\beta} = i_j. \tag{17}$$

The other equilibrium condition is

$$\tfrac{\gamma_j - \beta}{\beta} = \sigma\left[\tfrac{u'(q)}{c'(q)} - 1\right] \tag{18}$$

$\forall j \in \{1,2\}$, where (16) has been used.

Definition 1 *A symmetric steady state monetary equilibrium with two monies being accepted in each meeting is a couple* (q, i_j) *satisfying* (17)-(18).

At this point of the analysis the first main result can be introduced:

Proposition 1 *When two monies can be used for payment in each meeting, if they are accepted then they must grow at the same rate.*

Proof. Assume that both money 1 and money 2 can be used in each meeting. Then, (18) must hold, which implies

$$\tfrac{\gamma_1 - \beta}{\beta} = \tfrac{\gamma_2 - \beta}{\beta}$$

or, equivalently, $\gamma_1 = \gamma_2$. ■

Following Proposition 1, if two monies can be used for payment in each match, then their growth rate make all the difference. Therefore, money 1 and money 2 are symmetrically accepted if and only if they grow at the same rate.

4 Imperfect information

Until now, it has been assumed that both monies 1 and 2 can be accepted in decentralized meetings. In this section, this hypothesis will be relaxed by assuming that an agent can find himself in two types $j \in \{1,2\}$ of meetings in the DM: with probability α_1 he is in a meeting where only money 1 can be used for payment (type 1 meeting), while with probability $\alpha_2 = 1 - \alpha_1$ he can only use money 2 (type 2 meeting). This is formalized by introducing the assumption that agents are uninformed about the authenticity of money. Assume that each money type can be costlessly counterfeit and that some sellers – or each seller sometimes – cannot detect money j from counterfeits. Also assume that counterfeits perish after they change hands. This implies that agents will never accept the money type which authenticity cannot be detected. For now $\alpha_1 \in \mathbb{R}(0,1)$, and is exogenous; it will be endogenized below.

The value function of an agent who enters the centralized market with a portfolio $\mathbf{m}$ satisfies (7). The expected lifetime utility for an agent entering the DM with a portfolio $\mathbf{m}$ of monies is now

$$\begin{aligned} V(\mathbf{m}) = \sigma\alpha_1 & \{u(q_1) + W(\mathbf{m} - \mathbf{z}_{1,b}) - c(q_1) + W(\mathbf{m} + \mathbf{z}_{1,s})\} \\ & + \sigma\alpha_2 \{u(q_2) + W(\mathbf{m} - \mathbf{z}_{2,b}) - c(q_2) + W(\mathbf{m} + \mathbf{z}_{2,s})\} \\ & + (1 - 2\sigma) W(\mathbf{m}) \end{aligned} \tag{19}$$

where $\mathbf{z}_{1,b} \equiv (z_{1,b}, 0)$, $\mathbf{z}_{2,b} \equiv (0, z_{2,b})$, $\mathbf{z}_{1,s} \equiv (z_{1,s}, 0)$, and $\mathbf{z}_{2,s} \equiv (0, z_{2,s})$. The quantities q_1 and q_2 denote the units of good exchanged in a meeting of type 1 and type 2, respectively. Again, these quantities will be determined by Nash bargaining:

$$\max_{q_j, z_j} [u(q_j) - \phi_j z_j]^{\theta} [-c(q_j) + \phi_j z_j]^{1-\theta} \tag{20}$$

such that

$$z_j \leq m_j \tag{21}$$

$\forall j \in \{1, 2\}$, where $z_j = z_{j,b} = z_{j,s}$. Then, the solution to (20)-(21) is

$$\phi_j z_j = g(q_j) \equiv \frac{\theta u'(q_j)c(q_j) + (1-\theta)u(q_j)c'(q_j)}{\theta u'(q_j) + (1-\theta)c'(q_j)}. \tag{22}$$

Assuming that buyers have all the bargaining power, (22) reduces to $\phi_j z_j = g(q_j) = c(q_j)$. Again, if $\phi_j m_j \geq c(q_j^*)$, then the buyer exchanges $z_j < m_j$ of money j for the first best quantity q_j^*. Otherwise, he gives the seller all of his money j (i.e. $z_j = m_j$) in exchange for the quantity q_j that satisfies $c(q_j) = \phi_j m_j$. Thus, it holds that

$$q_j(\phi_j m_j) = \begin{cases} q_j^* & \text{if } \phi_j m \geq c(q_j^*) \\ c^{-1}(\phi_j m_j) & \text{if } \phi_j m_j < c(q_j^*). \end{cases} \tag{23}$$

Using the bargaining solution (23) and linearity of $W(\mathbf{m})$, the value function (19) can be rewritten as

$$\begin{aligned} V(\mathbf{m}) = {} & \sigma\alpha_1 \left[u\left(q_1\left(m_1\right)\right) - c\left(q_1\left(m_1\right)\right)\right] \\ & + \sigma\alpha_2 \left[u\left(q_2\left(m_2\right)\right) - c\left(q_2\left(m_2\right)\right)\right] + W(\mathbf{m}). \end{aligned} \tag{24}$$

Now, take the differential of (24) with respect to m_j for $j \in \{1, 2\}$ and get

$$V_j(\mathbf{m}) = W_j(\mathbf{m}) + \sigma\phi_j\alpha_j \left[\tfrac{u'(q_j)}{c'(q_j)} - 1\right] \tag{25}$$

where $\partial q_j / \partial m_j = \phi_j / c'(q_j)$ has been used. Again, from (9) and the second equation in (8) lagged one period one gets

$$\tfrac{\phi_{j,-1}}{\beta} = \phi_j \left\{1 + \sigma\alpha_j \left[\tfrac{u'(q_j)}{c'(q_j)} - 1\right]\right\}. \tag{26}$$

Then, using the Fisher equation and taking the steady state of (26), equilibrium conditions are

$$\tfrac{\gamma_j - \beta}{\beta} = i_j \tag{27}$$

and

$$\tfrac{\gamma_j - \beta}{\beta} = \sigma\alpha_j \left[\tfrac{u'(q_j)}{c'(q_j)} - 1\right] \tag{28}$$

$\forall j \in \{1, 2\}$.

Definition 2 *A symmetric steady state monetary equilibrium with imperfect information is a couple* (q_j, i_j) *satisfying* (27)-(28), $j \in \{1, 2\}$, *for given* α_j.

5 Endogenous acceptability

In the previous section, the seller's decision to accept money 1 or money 2 was exogenous. It will be now endogenized. In doing so, the assumption that agents can accept only one money type for payment will be retained, while the hypothesis that they are uninformed will be relaxed. Before deriving conditions for voluntary acceptability, note that the probability that a seller accepts money j (α_j) depends on the value of money j; but the value of money j depends on the probability that other sellers accept money j. Let us denote this probability by $\alpha_{j,-s}$. In a symmetric equilibrium, α_j must be equal to $\alpha_{j,-s}$.

The seller's expected benefit of accepting money 1 in the DM is given by the extra utility from being in a type 1 meeting, as opposed to a type 2 meeting, i.e.

$$\Delta(\alpha_1) \equiv [g(q_1(\alpha_1)) - c(q_1(\alpha_1))] - [g(q_2(\alpha_1)) - c(q_2(\alpha_1))]. \qquad (29)$$

From the bargaining solution (22), it holds that $g(q_j) = c(q_j)$ if $\theta = 1$; note that if the seller's bargaining power is zero, his net benefit is zero in each type j of meeting for any q_j.

Lemma 1 *$\theta = 1$ is a sufficient condition for $\Delta(\alpha_1) \equiv 0$.*

Proof. Assume $\theta = 1$. Then, by (22), $g(q_j) = c(q_j)$ for any j, which implies $\Delta(\alpha_1) \equiv 0$. ■

Now, assume that sellers have a strictly positive bargaining power – i.e. $\theta < 1$. Then, $g(q_j) > c(q_j)$ holds directly from (22). So that: (i) $\Delta(\alpha_1) \equiv 0$ iff $q_1 = q_2$, (ii) $\Delta(\alpha_1) > 0$ iff $q_1 > q_2$, and (iii) $\Delta(\alpha_1) < 0$ iff $q_1 < q_2$. (This is a direct consequence of the fact that $\theta < 1$ implies $g'(q) > c'(q)$ for any $q < q^*$.)

To summarize, if $\theta < 1$ and information is perfect, then there can be three distinct equilibria (Figure 3). (i) *Only money 1 is accepted*: $\Delta(\alpha_1) > 0$ implies $\alpha_1 = 1$, thus all sellers accept money 1 for payment with probability 1, and money 2 with probability 0 (top right point). In this equilibrium, nobody brings money 2 into the DM. (ii) *Only money 2 is accepted*: $\Delta(\alpha_1) < 0$ implies $\alpha_1 = 0$ (bottom left point). Thus money 2 is accepted for payment with probability 1, and money 1 with probability 0. In this equilibrium, nobody has the incentive to carry money 1 out of the CM. (iii) *Dual currency circulation*: $\Delta(\alpha_1) \equiv 0$ implies that sellers accept money 1 with probability $\alpha_1 \in \mathbb{R}(0,1)$, while they accept money 2 with probability $1-\alpha_1$ (point along the diagonal between the two).

Now, use $\alpha_2 = 1 - \alpha_1$ into (28) and rearrange to get

$$\alpha_1 = \frac{(\gamma_1-\beta)\left[\frac{u'(q_2)}{c'(q_2)}-1\right]}{(\gamma_2-\beta)\left[\frac{u'(q_1)}{c'(q_1)}-1\right]+(\gamma_1-\beta)\left[\frac{u'(q_2)}{c'(q_2)}-1\right]}. \qquad (30)$$

To determine α_1 such that $\Delta(\alpha_1) \equiv 0$, use $q_1 = q_2$ into (30) and get

$$\alpha_1 = \frac{\gamma_1-\beta}{\gamma_1+\gamma_2-2\beta}. \qquad (31)$$

Figure 3: Best Response Correspondence

Equation (31) determines value of α_1, given γ_1 and γ_2, such that dual currency circulation holds, i.e. such that sellers are indifferent between accepting the foreign money or accepting the local one.

The following Lemma can now be introduced:

Lemma 2 *If $\theta < 1$, information is perfect, and $\Delta(\alpha_1) \equiv 0$, then α_1 is increasing in γ_1 and decreasing in γ_2. The opposite holds for α_2.*

Proof. Take the differential of (31) with respect to γ_1 and γ_2, and obtain

$$\frac{\partial \alpha_1}{\partial \gamma_1} = \frac{\gamma_2 - \beta}{(\gamma_1 + \gamma_2 - 2\beta)^2} > 0$$

$$\frac{\partial \alpha_1}{\partial \gamma_2} = -\frac{\gamma_1 - \beta}{(\gamma_1 + \gamma_2 - 2\beta)^2} < 0,$$

respectively. Using $\alpha_1 = 1 - \alpha_2$, the inequalities $\partial \alpha_2 / \partial \gamma_2 > 0$ and $\partial \alpha_2 / \partial \gamma_1 < 0$ are proved along the same lines. ■

The next main result can be established:

Proposition 2 *In a dual currency circulation economy: (i) the local money circulates more widely than the foreign money, (ii) local and foreign monies are symmetrically accepted if and only if they grow at the same rate.*

Proof. Assume that only one money can be used in a meeting. Also assume $\theta < 1$, perfect information, and $\Delta(\alpha_1) \equiv 0$. Then (31) must hold. By Lemma 2, this implies that: (i) $\alpha_1 > \alpha_2$ iff $\gamma_1 > \gamma_2$, and $\alpha_1 < \alpha_2$ iff $\gamma_1 < \gamma_2$, and (ii) $\alpha_1 = \alpha_2 = 1/2$ iff $\gamma_1 = \gamma_2$. ■

The foreign money provides a better protection against the inflation tax, but is less accepted for payment in the DM. The local money has an higher acceptability value, but provides a less protection against the inflation. Thus, in a dual currency circulation economy, the local money (i.e. a money with a larger γ) should circulate more widely than a foreign money (i.e. a money with a smaller γ). This is because sellers accept the local money for payment more frequently than they accept the foreign one, which is necessary to induce buyers to take the local money out of the CM. The local money provides less protection against the inflation tax so buyers want to spend it as soon as they can.

6 Conclusions

[To be added]

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