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Endogenous Currencies Acceptability

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Abstract

This paper studies currencies acceptability in an economy *à la* Lagos and Wright [20]. Monies play an essential role as media of exchange in decentralized markets flawed by frictions, and an asset-like role in centralized markets. When monies are allowed to compete, the acceptability of each depends (i) on its ability to provide insurance against the inflation tax in centralized markets and (ii) on its role as a means of payment in decentralized markets. When monies growth rates differ, the first effect disincentives demand of the more intensively issued money, which holders sellers expect to come across more likely in decentralized markets. A tradeoff between less inflation tax and fewer chances of selling goods provides new insights into the characterization of dual currency regime equilibria. As agents attempt at balancing off the sum of expected returns from using each money both as an asset and as a means of payment, the following outcomes are possible: (i) the best asset drives the worst asset out of circulation when the first effect dominates; (ii) monies coexist when expected returns are equal across currencies and (iii) the worst asset drives the best one out of circulation when the protection effect against missed sales provided in decentralized markets is large enough to cause a sellers bias towards acceptability of the more intensively issued currency up to domination of the first effect. In this latter instance, and unlike models without competing media of exchange, deflating the best asset in excess of the Friedman rule is consistent with monetary equilibrium. The optimal monetary policy consists of deflating the best asset (the worst asset) in excess (short) of the Friedman rule by an amount equal to its liquidity gap (premium). Under purchasing power parity, this results in an ever appreciating exchange rate. The framework of analysis can be extended to geographically separated decentralized markets and an international centralized market. On the former market agents may need to exchange foreign monies for domestic,

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while only monies' purchasing power matters in the centralized market. Demand and supply for monies in the decentralized market determines spot exchange rates which may temporarily deviate from trends consistent with PPP. The relationship between monetary and exchange rate policy, along with their effects on local and international welfare, should be studied.

Keywords: Fiat money, endogenous currencies acceptability, imperfect information, search and matching

JEL Classification: D8, E4, E5

“A long-held notion is that an inferior currency should circulate more widely than a superior money. Those holding both monies would prefer spending the ‘bad’ money as soon as they can, and keep the ‘good’ money for future purchases;” Camera, Craig and Waller [4], p..

1 Introduction

In many South American and Eastern European Countries, citizens adopt a dual-payment system by using the foreign currency – the dollar or the euro – in addition to their own domestically issued currency as a means of payment. In these countries it is observed that the foreign currency is not universally accepted, however, in the sense that the great majority of agents make use of the local currency in transactions. This phenomenon rises at least three questions. First, under what conditions do the foreign and domestic currencies co-exist? Second, when is the domestic currency more widely accepted than the foreign one and why? Third, what is the role of monetary policy in each case? The focus of this paper is to address these questions in a fundamental model of money.

New generation (microfounded) models of monetary economics have become the natural framework to study dual currency acceptability as they formalize the essential role of money explicitly, not by assumption. A first wave of search-money literature on multiple currencies includes [1], [7], [8], [13], [24], [28], [30] and many others. All these papers share the assumption that money is an indivisible object, and individual holdings of money are bounded at one unit (first- and second-generation models). An exhaustive survey of these models is in [5]. Some attempts to study multiple-money holdings in a two-currency setting are [4], [6], and [11]. In [4], agents are allowed to hold two units of indivisible money. [11] relax the restriction of money indivisibility, but they assume that goods are indivisible. In their paper, [6] derive numerical results within a both divisible-money and divisible-good setup.

This paper characterizes dual currencies acceptability by relaxing (i) money indivisibility and (ii) boundedness of money holding to one unit, while retaining analytical tractability inherited by third-generation search monetary models that permits to asses the effects of monetary policy on currencies circulation.

The papers which are closest to ours are Geromichalos, Licari, and Suárez-Lledó [10], Lagos [15], Lagos and Rocheteau [17], Lester, Postlewaite, and Wright [21], Lester, Postlewaite, and Wright [22] and Shi [27]. Lagos and Rocheteau [17] discuss a model where capital is accepted as a medium of exchange and agents may be led to over-accumulate it in excess of the modified golden rule level

when the stock of real assets is too low to sustain efficient trade. In this case, money improves intertemporal welfare by reducing capital overaccumulation. Lester, Postlewaite, and Wright [21] study the effect of monetary policy on prices and allocations in a model with money and alternative assets. They show that the prices of these assets are potentially affected by the rate of return on money. When the rate of return of money falls agents substitute out of currency and into less liquid assets and vice versa. In a different paper, Shi [27] assumes that agents can use either money or nominal bonds to acquire some goods, but they are not allowed to use bonds as payment for other goods. He shows that legal restrictions are welfare improving. Our model departs from these papers in two way. First, we endogenously derive to which extent each currency is accepted as a medium of exchange. Second, we discuss how monetary policy affects each currency acceptability.

A number of papers are closely related to what we do here (i.e. Devereux and Shi [9], Head and Shi [12], and Liu and Shi [23]). Devereux and Shi [9] study a model in which a currency can arise endogenously as an internationally accepted medium of exchange, facilitating cross-country trade, and allowing agents to economize on resources when trading currencies requires large transaction costs. They show that a vehicle currency reduces the cost of currency trade by eliminating the need of setting up bilateral currency trading posts among all possible countries. Head and Shi [12] study a two-country model to determine the nominal exchange rate between two currencies. They show that the equilibrium nominal exchange rate reflects the countries' economic fundamentals, including the growth rate of monies. In a similar paper, Liu and Shi [23] study the coordination effort between two currency areas in setting long-run inflation. They show that monetary coordination reduces inflation, increases consumption, and improves welfare. As in our model, these papers also get a degenerate distribution of money holdings across agents but by different means. The differs from our work in that they assume that the fundamental decision-making unit is a household with a continuum of agents (Shi [26]).

The main result of this paper is the following. In a dual currency economy, (i) the money type with a higher rate of growth is more widely accepted than the money type with a lower rate of growth, and (ii) unbiased acceptability between the two monies holds if and only if they grow at the same rate. That is, in a dual currency economy with asymmetric acceptability the money with a lower rate of growth provides a better protection against the inflation tax, but is less accepted for payment in the DM. Conversely, the money with a higher rate of growth has a higher acceptability value, but provides less protection against the inflation.

The paper is organized as follows. Section 2 describes the basic framework and the agents'

decision problem. Stationary equilibria partial acceptability are characterized in Section 3. Section 4 derives conditions for endogenous acceptability. Section 5 extends the model to include information costs in the analysis. The conclusions end the paper.

2 The model

The basic setup is borrowed from LW. Time is indexed by $t \in \mathbb{N}$ and each period t is divided into two subperiods where different activities take place. There is a $[0, 1]$ *continuum* of infinitely-lived agents and two types of perfectly divisible commodities – general and special goods. Each agent produces a subset distinct from the subset of special goods he consumes. Specialization is modeled as follows. In the first subperiod, each agent meets someone who produces a good he wishes to consume with probability $\sigma \in (0, 1/2]$ and meets someone who likes the good he produces with the same probability σ . With probability $1 - 2\sigma$ an agent has no opportunity to trade. Let us denote consumers as buyers and producers as sellers. The specialization of agents over consumption and production of the special good gives rise to a ‘double coincidence of wants’ problem. In contrast to special goods, general goods can be consumed and produced by all agents.

Special goods can only be produced during the first subperiod, while general commodities can only be produced during the second subperiod. In the first subperiod, agents participate in a decentralized market (DM) where each meeting is bilateral and is a random draw from the set of pairwise meetings. In this market the terms of trade are determined by a Nash bargaining protocol. In the second subperiod agents produce general goods and can trade in a centralized market (CM).

Agents get utility $u(q)$ from q consumption in the DM, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $u(0) = 0$. It is assumed that the elasticity of utility $e(q) = qu'(q)/u(q)$ is bounded. Producers incur utility cost $c(q)$ from producing q units of output with $c'(q) > 0$, $c''(q) \geq 0$, and $c(0) = 0$. Let q^* denote the solution to $u'(q^*) = c'(q^*)$.

In the CM all agents consume and produce, getting utility $U(x)$ from x consumption, with $U'(x) > 0$, $U'(0) = \infty$, $U'(\infty) = 0$, $U''(x) \leq 0$ and $U(0) = 0$. Let x^* be the solution to $U'(x^*) = 1$. All agents can produce consumption goods from labor using a linear technology. Agents discount between the CM and the next-period DM, but not between DM and CM. This is not restrictive since as in [25] all that matters is the total discounting between one period and the next. It is assumed that individual actions are not observable in the CM so as to avoid contagion equilibria (see Aliprantis, Camera Puzzello [2, 3]).

Figure 1: Timing of events

Figure 2: Environment

All agents are *anonymous* in the DM and there is a complete lack of commitment. Consequently, trade credit is ruled out and transactions are subject to a *quid pro quo* restriction, so there is a role for a medium of exchange (Kocherlakota [14] and Wallace [29]).

Let us analyze the first best allocation. At the beginning of a period, the expected steady state lifetime utility of the representative agent is

$$(1 - \beta) \mathcal{W} = \sigma [u(q) - c(q)] + U(x) - x \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor and q the quantity of goods consumed by a buyer and produced as a seller in the DM. The solution to the planner's problem in an economy without anonymity yields

$$U'(x^*) = 1, \quad (2)$$

$$u'(q^*) = c'(q^*). \quad (3)$$

These are the quantities chosen by a social planner who could force agents to produce and consume.

There are two types $j = d, f$ of durable and intrinsically useless objects: money h and money f . It is assumed that two independent central banks exist that control the supply of each money type at any time t , $M_{j,t} > 0$. It is also assumed that $M_{j,t} = \gamma_j M_{j,t-1}$, where $\gamma_j > 0$ is constant and new money of type j is injected, or withdrawn if $\gamma_j < 1$, as lump-sum transfers $\pi_j M_{j,t-1} = (\gamma_j - 1) M_{j,t-1}$ to all agents. We restrict our analysis to policies where the domestic money grows at a rate greater than or equal to the rate of growth of the foreign money, $\gamma_d \geq \gamma_f \geq \beta$. The time subscript t is omitted and shorten $t + 1$ to $+$, $t - 1$ to $-$, etc. in what follows.

The timing of events is shown in Figure 1. At the beginning of the DM, bilateral trade of goods begins. In the CM agents receive lump sum transfers, produce, consume and rebalance their money holdings. The structure of this economy is shown in Figure 2.

At time t , let $\phi_j = 1/P_j$ be the real price of money j and P_j the price of goods in term of money j in the CM. The focus here is on symmetric steady state equilibria, where aggregate real money

balances are constant over time, i.e.

$$\begin{aligned}\phi_d M_d &= \phi_{d,-} M_{d,-}, \text{ and} \\ \phi_f M_f &= \phi_{f,-} M_{f,-}\end{aligned}\tag{4}$$

where $\phi_{j,-}/\phi_j = M_j/M_{j,-} = \gamma_j$.

In what follows we focus on a generic period t and work backwards from the CM to the DM.

Let $V(\mathbf{m})$ and $W(\mathbf{m})$ denote the expected value from trading in the DM, respectively the CM, with a portfolio $\mathbf{m} \equiv (m_d, m_f)$, where m_j denotes the amount of money j . In the CM agents produce h units of good using h hours of labor, receive lump sum transfers, consume, and adjust their money balances. Hence, the representative agent's problem in the CM is

$$W(\mathbf{m}) = \max_{x, h, \mathbf{m}_+} [U(x) - h + \beta V_+(\mathbf{m}_+)]\tag{5}$$

such that

$$x + \phi \mathbf{m}_+ = h + \phi \mathbf{m} + \phi \mathbf{T}\tag{6}$$

where the vector $\mathbf{m}_+ \equiv (m_{d,+}, m_{f,+})$ is the portfolio of monies taken into period $t+1$, $\phi \equiv (\phi_d, \phi_f)$ the real price of monies, and $\mathbf{T} \equiv (\pi_d M_{d,-}, \pi_f M_{f,-})$ the lump sum transfers. Eliminate h from (5) using (6) and get

$$W(\mathbf{m}) = \phi [\mathbf{m} + \mathbf{T}] + \max_{x, \mathbf{m}_+} [U(x) - x - \phi \mathbf{m}_+ + \beta V_+(\mathbf{m}_+)]\tag{7}$$

The first order conditions (FOCs) with respect to x and \mathbf{m}_+ are

$$\begin{aligned}U'(x) &= 1, \\ \beta V_{d,+}(\mathbf{m}_+) &= \phi_d, \text{ and} \\ \beta V_{f,+}(\mathbf{m}_+) &= \phi_f\end{aligned}\tag{8}$$

where the term $\beta V_{j,+}(\mathbf{m}_+)$ is the marginal benefit of taking money j out of the CM and ϕ_j is its marginal cost.

Two comments are in order from (8). First, the quantity of goods x consumed by every agent is equal to the efficient level x^* , where $U'(x^*) = 1$. Second, \mathbf{m}_{+1} is independent of \mathbf{m} . As a result, the portfolio of monies is degenerate at the beginning of the following period. This is due to the

quasi-linearity assumption in (5), which eliminates the wealth effects on money demand in the CM; agents who bring too much money (of type d or f) into the CM spend some buying goods, while those with too little cash sell goods. We do not assume a market for direct currency exchange. If there were a market in which agents can trade the two currencies then the market clearing price would be ϕ_d/ϕ_f .¹

The envelope conditions are

$$\begin{aligned} W_d(\mathbf{m}) &= \phi_d, \text{ and} \\ W_f(\mathbf{m}) &= \phi_f \end{aligned} \tag{9}$$

where $W_j(\mathbf{m})$ is the derivative of $W(\mathbf{m})$ with respect to m_j . An agent who has a portfolio \mathbf{m} of monies at the opening of the DM has expected lifetime utility

$$V(\mathbf{m}) = \sigma [u(q) + W(\mathbf{m} - \mathbf{z}_b)] + \sigma [-c(q) + W(\mathbf{m} + \mathbf{z}_s)] + (1 - 2\sigma) W(\mathbf{m})$$

where $\mathbf{z}_b \equiv (z_{1,b}, z_{2,b})$ is the amount of monies given up when a buyer and $\mathbf{z}_s \equiv (z_{1,s}, z_{2,s})$ the amount of monies received as a seller. From linearity of $W(\mathbf{m})$, expression (7) can be rewritten as

$$W(\mathbf{m}) \equiv W(0) + \phi \mathbf{m}$$

which can be used to rewrite the indirect utility function as follows

$$V(\mathbf{m}) = W(\mathbf{m}) + \sigma [u(q) - \phi \mathbf{z}_b] + \sigma [-c(q) + \phi \mathbf{z}_s]. \tag{10}$$

Again, due to linearity of $W(\mathbf{m})$, the Nash bargaining problem in the DM reduces to

$$\max_{q, \mathbf{z}} [u(q) - \phi \mathbf{z}]^\theta [-c(q) + \phi \mathbf{z}]^{1-\theta}$$

such that

$$\mathbf{z} \leq \mathbf{m} \tag{11}$$

where $\theta \in \mathbb{R}(0, 1]$ is the buyer's bargaining power, and $\mathbf{z} = \mathbf{z}_b = \mathbf{z}_s$ the portfolio of monies exchanged in a bilateral meeting. The constraint (11) means that buyers cannot spend more monies than what

¹A market for direct currency exchanges is inessential here in the sense an agent can rebalance his portfolio for monies by money-goods trades in the CM. As pointed out by Head and Shi [12], the existence of a frictionless CM market for goods and linearity of production function both imply no discrepancies between the relative prices in the two markets (the CM and the direct currency exchange market).

they bring into the DM. The solution to the bargaining problem is

$$\phi \mathbf{z} = g(q) \equiv \frac{\theta u'(q)c(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)} \quad (12)$$

where $g(q) > 0$. If $\phi \mathbf{m} \geq g(q^*)$ then the buyer exchanges $\mathbf{z} < \mathbf{m}$ of his portfolio for the first best quantity q^* . Otherwise, he gives the seller all of his portfolio, $\mathbf{z} = \mathbf{m}$, in exchange for the quantity q that satisfies $g(q) = \phi \mathbf{m}$. The outcome is independent of the seller's portfolio of monies, and it depends on \mathbf{m} only if the constraint $\mathbf{z} \leq \mathbf{m}$ binds.

Thus, it holds that

$$q(\mathbf{m}) = \begin{cases} q^* & \text{if } \phi \mathbf{m} \geq g(q^*), \\ g^{-1}(\phi \mathbf{m}) & \text{if } \phi \mathbf{m} < g(q^*) \end{cases} \quad (13)$$

then, taking the differential of (10) with respect to m_j for $j \in \{1, 2\}$ and using the bargaining solution (13), yields

$$\begin{aligned} V_d(\mathbf{m}) &= W_d(\mathbf{m}) + \phi_d \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right], \text{ and} \\ V_f(\mathbf{m}) &= W_f(\mathbf{m}) + \phi_f \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right] \end{aligned} \quad (14)$$

where $\partial q / \partial m_j = \phi_j / g'(q)$ has been used. (Notice that if (11) is binding, then $\phi \mathbf{m} = g(q)$.)

By (14), (9), and the second condition in (8) lagged one period, the following holds

$$\begin{aligned} \frac{\phi_{d,-}}{\beta} &= \phi_d \left\{ 1 + \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right] \right\}, \text{ and} \\ \frac{\phi_{f,-}}{\beta} &= \phi_f \left\{ 1 + \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right] \right\} \end{aligned} \quad (15)$$

Now, replace $\phi_{j,-} / \phi_j$ with γ_j and rewrite (15) as

$$\begin{aligned} \frac{\gamma_d - \beta}{\beta} &= \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right], \text{ and} \\ \frac{\gamma_f - \beta}{\beta} &= \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right] \end{aligned} \quad (16)$$

which is satisfied if and only if $\gamma_d = \gamma_f = \gamma$. Since domestic and foreign currencies are equally accepted as media of exchange –i.e. they have the same liquidity value– they must grow at the same rate in order for agents to hold both types of monies in their portfolio. Using $\gamma_d = \gamma_f$ we can write (16) as

$$\frac{\gamma - \beta}{\beta} = \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right] \quad (17)$$

Definition 1 *A symmetric steady state monetary equilibrium in which two monies are equally*

accepted in each meeting is a quantity q satisfying (17) given parameters (β, σ, γ) .

So far, we have assumed that both currencies can be used for goods purchases in decentralized meetings. We now relax this hypothesis by introducing the restriction that an agent can find himself in two types of meetings in the DM: (i) he is in a meeting where only the domestic money can be used for payment, with probability α_1 , and (ii) he can use both currencies, with probability $\alpha_2 \equiv 1 - \alpha_1$.

In doing so, we introduce an information problem as follows. We assume that the foreign currency, but not the domestic currency, can be costlessly counterfeited by agents. Counterfeits perish after they change hands. Before bilateral trades take place, agents receive an idiosyncratic shock such that they are *informed* about genuinity of the foreign money with probability α_2 , and *uninformed* with the complement probability α_1 . Informed agents can perfectly and costlessly recognize the authenticity of the foreign currency, while those who are uninformed cannot tell the difference between counterfeit and genuine foreign currency.

The agent's ability (or inability) to detect genuine foreign money is public knowledge. This assumption assures that agents who are uninformed will never accept it for payment in the DM. To see this, consider a meeting between a buyer and an uninformed seller. Since the buyer knows that his partner is uninformed, he will only offer him counterfeit foreign money in exchange for goods. But the seller anticipates this, so she will refuse to take anything other than the domestic money for payment.² For now $\alpha_1 \in (0, 1)$, and is exogenous; it will be endogenized below.

The value function of an agent who enters the centralized market with a portfolio \mathbf{m} satisfies (7). The expected lifetime utility for an agent entering the DM with a portfolio \mathbf{m} of monies is now

$$\begin{aligned} V(\mathbf{m}) = & \sigma\{\alpha_1[u(q_1) + W(\mathbf{m} - \mathbf{z}_{1,b})] + (1 - \alpha_1)[u(q_2) + W(\mathbf{m} - \mathbf{z}_{2,b})]\} \\ & + \sigma\{\alpha_1[-c(q_1) + W(\mathbf{m} + \mathbf{z}_{1,s})] + (1 - \alpha_1)[-c(q_2) + W(\mathbf{m} + \mathbf{z}_{2,s})]\} \\ & + (1 - 2\sigma)W(\mathbf{m}) \end{aligned} \quad (18)$$

where $\mathbf{z}_{1,b} \equiv (z_{1d,b}, 0)$, $\mathbf{z}_{2,b} \equiv (z_{2d,b}, z_{2f,b})$, $\mathbf{z}_{1,s} \equiv (z_{1d,s}, 0)$, and $\mathbf{z}_{2,s} \equiv (z_{2d,s}, z_{2f,s})$. The quantities q_1 and q_2 denote the units of good exchanged in a meeting when the seller is uninformed and informed, respectively. As before, we assume that these quantities will be determined by Nash

²Alternatively, partial acceptability of domestic money can be achieved through *government transaction policies*, by assuming the existence of an exogenous fraction α_1 of government agents that accept the domestic money only.

bargaining as follows. In a type-1 meeting, the terms of trade (q_1, \mathbf{z}_1) solve:

$$\max_{q_1, \mathbf{z}_1} \{u(q_1) - \phi \mathbf{z}_1\}^\theta \{-c(q_1) + \phi \mathbf{z}_1\}^{1-\theta} \quad (19)$$

such that

$$\mathbf{z}_1 \leq \mathbf{m} \quad (20)$$

where $\mathbf{z}_1 = \mathbf{z}_{1,b} = \mathbf{z}_{1,s} = (z_{1d}, 0)$. The solution to (19)-(20) is

$$\phi \mathbf{z}_1 = \phi_d z_{1d} = g(q_1) \equiv \frac{\theta u'(q_1)c(q_1) + (1-\theta)u(q_1)c'(q_1)}{\theta u'(q_1) + (1-\theta)c'(q_1)} \quad (21)$$

where $z_{1d} = z_{1d,b} = z_{1d,s}$. If $\phi_d m_d \geq g(q_1^*)$ then the buyer exchanges $z_{1d} < m_d$ of the domestic money for the first best quantity q_1^* . Otherwise, he gives the seller all of his domestic money (i.e. $z_{1d} = m_d$) in exchange for the quantity q_1 that satisfies $g(q_1) = \phi_d m_d$. Thus, it holds that

$$q_1(m_d) = \begin{cases} q_1^* & \text{if } \phi_d m_d \geq g(q_1^*) \\ g^{-1}(\phi_d m_d) & \text{if } \phi_d m_d < g(q_1^*). \end{cases} \quad (22)$$

In a type-2 meeting, the couple (q_2, \mathbf{z}_2) must satisfy

$$\max_{q_2, \mathbf{z}_2} \{u(q_2) - \phi \mathbf{z}_2\}^\theta \{-c(q_2) + \phi \mathbf{z}_2\}^{1-\theta} \quad (23)$$

such that

$$\mathbf{z}_2 \leq \mathbf{m} \quad (24)$$

where $\mathbf{z}_2 = \mathbf{z}_{2,b} = \mathbf{z}_{2,s} = (z_{2d}, z_{2f})$. The bargaining problem (23)-(24) yields the following solution

$$\phi \mathbf{z}_2 = \phi_f z_{2d} + \phi_f z_{2f} = g(q_2) \equiv \frac{\theta u'(q_2)c(q_2) + (1-\theta)u(q_2)c'(q_2)}{\theta u'(q_2) + (1-\theta)c'(q_2)}. \quad (25)$$

If $\phi_d m_d + \phi_f m_f \geq g(q_2^*)$, then the buyer spends $z_{2d} < m_d$ of the domestic money and $z_{2f} < m_f$ of the foreign money to purchase the efficient quantity q_2^* . Otherwise, he gives the seller all of portfolio of monies, $z_{2d} = m_d$ and $z_{2f} = m_f$, in exchange for the quantity q_2 that satisfies $g(q_2) = \phi_d m_d + \phi_f m_f$.

Hence, in a type-2 meeting, it holds that

$$q_2(m_d, m_f) = \begin{cases} q_2^* & \text{if } \phi_d m_d + \phi_f m_f \geq g(q_2^*) \\ g^{-1}(\phi_d m_d + \phi_f m_f) & \text{if } \phi_d m_d + \phi_f m_f < g(q_2^*). \end{cases} \quad (26)$$

We are now able to derive the marginal value of monies. To do so, use the bargaining solutions (22) and (26), linearity of $W(\mathbf{m})$, and take the differential of (18) with respect to m_d and m_f to obtain

$$V_d(\mathbf{m}) = \sigma \phi_d \left\{ \alpha_1 \left[\frac{u'(q_1)}{g'(q_1)} - 1 \right] + (1 - \alpha_1) \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right] \right\} + W_d(\mathbf{m}) \quad (27)$$

and

$$V_f(\mathbf{m}) = \sigma \phi_f (1 - \alpha_1) \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right] + W_f(\mathbf{m}) \quad (28)$$

where $\partial q_1 / \partial m_d = \phi_d / g'(q_1)$, $\partial q_1 / \partial m_f = 0$, $\partial q_2 / \partial m_d = \phi_d / g'(q_2)$, and $\partial q_2 / \partial m_f = \phi_f / g'(q_2)$ have been used. By virtue of (9), and the second condition in (8) lagged one period, expressions (27) and (28) can be rewritten as

$$\frac{\gamma_d - \beta}{\beta} = \sigma \left\{ \alpha_1 \left[\frac{u'(q_1)}{g'(q_1)} - 1 \right] + (1 - \alpha_1) \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right] \right\} \quad (29)$$

and

$$\frac{\gamma_f - \beta}{\beta} = \sigma (1 - \alpha_1) \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right]. \quad (30)$$

Definition 2 *A symmetric steady state monetary equilibrium with partial acceptability is a time invariant path (q_1, q_2) satisfying (29)-(30) given parameters $(\beta, \sigma, \gamma_d, \gamma_f, \alpha_1)$.*

3 Endogenous acceptability

In the previous section, foreign currency acceptability was exogenous. It will be now endogenized. To do so, we impose the condition that sellers are indifferent between accepting the domestic or the foreign currency, and solve for α_2 .

Before deriving conditions for voluntary acceptability, it must be noted that the probability of a representative seller accepting the foreign money, α_2 , depends on how valuable the foreign money is which, in turn, depends on other sellers' beliefs about acceptance of the foreign currency in transactions, $\hat{\alpha}_2$. In a symmetric equilibrium, it must hold that $\alpha_2 = \hat{\alpha}_2$. We anticipate the result

that asymmetric monetary growth selects symmetric equilibria where the fastest growing money is the most widely accepted as a means of payment by agents in the DM.

The seller's net expected benefit of accepting the foreign currency in the DM is given by the extra utility from being in a type-2, as opposed to a type-1, meeting, i.e.

$$\Delta(\alpha_2) \equiv g[q_2(\alpha_2)] - c[q_2(\alpha_2)] - g[q_1(\alpha_2)] + c[q_1(\alpha_2)]. \quad (31)$$

Now, if $\theta < 1$, then $g(q_j) > c(q_j)$ directly from (21), and three outcomes are possible: (i) $\Delta(\alpha_2) = 0 \Leftrightarrow q_1 = q_2$ (due to monotonicity of the seller's surplus, there is only one value of q that satisfies $\Delta(\alpha_2) = 0$); (ii) $\Delta(\alpha_2) > 0 \Leftrightarrow q_1 < q_2$; and (iii) $\Delta(\alpha_2) < 0 \Leftrightarrow q_1 > q_2$.

Thus, there can be three Nash monetary equilibria when $\theta < 1$: two in pure strategies and one in mixed strategies (Figure 3). (i) In the first case the foreign money is not accepted by any seller. If the seller's net benefit of accepting both monies is negative, $\Delta(\alpha_2) < 0$, then he will accept the domestic currency and not the foreign currency, $\alpha_2 = 0$. This means that all sellers accept the domestic currency for payment, and nobody accepts the foreign currency (bottom left point). In this equilibrium, there is no incentive for anybody to carry the foreign money into the DM. Consequently, the domestic money is the only medium of exchange. (ii) In the second case the foreign money is accepted by each seller. If the seller's net benefit of accepting both currencies is strictly positive, $\Delta(\alpha_2) > 0$, then he will accept any money in a meeting, $\alpha_2 = 1$ (top right point). That is, the domestic and the foreign currency are jointly accepted for payment by everyone –both currency types are symmetrically accepted in transactions, that is they have the same liquidity value. Hence, everybody has the incentive to carry both monies out of the CM. (iii) In the last case the foreign money is partially accepted in the sense that some sellers end up accepting only the domestic money while the others accept both currencies. This is because the seller's net benefit of accepting both currencies in a meeting is zero, $\Delta(\alpha_2) = 0$, which means that he is indifferent between being in a type-1 or type-2 meeting. In a symmetric equilibrium sellers randomize between accepting only the domestic currency, with probability α_1 , and accepting both currencies, with probability α_2 (point along the diagonal between the two). In this equilibrium the domestic currency is more widely accepted than the foreign currency.

Note that if the seller's bargaining power is zero (i.e. $\theta = 1$) then $g(q_j) = c(q_j)$ for any q_j , $j \in \{1, 2\}$. If $\theta = 1$, then buyers extract all the surplus from trade, so the seller's net benefit is always zero, in each type of meeting and for any quantity traded. Thus, $\theta = 1$ is a sufficient

Figure 3: Best Response Correspondence

condition for $\Delta(\alpha_2) = 0$.

Let us define the equilibrium for $\theta < 1$:

Definition 3 *A single currency equilibrium is a path (q_1, q_2) satisfying (29)-(30), and $\Delta(\alpha_2) < 0$. A dual currency equilibrium with symmetric acceptability is a path (q_1, q_2) satisfying (29)-(30), and $\Delta(\alpha_2) > 0$. A dual currency equilibrium with asymmetric acceptability is a path (q_1, q_2) satisfying (29)-(31) and $\Delta(\alpha_2) = 0$.*

Using (29) and (30) and rearranging terms, one gets

$$\alpha_2 = \frac{(\gamma_f - \beta) \left[\frac{u'(q_1)}{g'(q_1)} - 1 \right]}{(\gamma_f - \beta) \left[\frac{u'(q_1)}{g'(q_1)} - \frac{u'(q_2)}{g'(q_2)} \right] + (\gamma_d - \beta) \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right]} \quad (32)$$

Given our assumptions about $u(q)$ and $c(q)$, there is only one value of $q \equiv q_1 = q_2$ such that $\Delta(\alpha_2) = 0$ holds. Substituting q into (32) yields

$$\alpha_2 = \frac{\gamma_f - \beta}{\gamma_d - \beta}. \quad (33)$$

Equation (33) determines the value of α_2 such that differential acceptability holds, i.e. such that sellers are indifferent between accepting both monies and accepting only the domestic currency.

The next statement can now be introduced:

Lemma 1 *If $\theta < 1$ and $\Delta(\alpha_2) = 0$, then α_2 is decreasing in γ_d and increasing in γ_f . The opposite holds for α_1 .*

Proof. Take the differential of (33) with respect to γ_d and γ_f , and obtain

$$\frac{\partial \alpha_2}{\partial \gamma_d} = -\frac{\gamma_f - \beta}{(\gamma_d - \beta)^2} < 0$$

$$\frac{\partial \alpha_2}{\partial \gamma_f} = \frac{1}{(\gamma_d - \beta)} > 0,$$

respectively. Using $\alpha_1 = 1 - \alpha_2$, the inequalities $\partial \alpha_1 / \partial \gamma_d > 0$ and $\partial \alpha_1 / \partial \gamma_f < 0$ are proved along the same lines. ■

The main result of the paper can now be established:

Figure 4: Dual currency economy

Proposition 1 *The following holds:*

(i) *in a dual currency equilibrium with symmetric acceptability the domestic and the foreign money grow at the same rate;*

(ii) *in a dual currency equilibrium with asymmetric acceptability the more widely accepted money must grow at a faster rate.*

Proof. Statement (i). Assume dual currency equilibrium with symmetric acceptability ($\Delta(\alpha_2) > 0$). This means that the foreign is fully accepted for payment, i.e. $\alpha_2 = 1$. By virtue of (29) and (30), this implies $\gamma_d = \gamma_f$.

Statement (ii). Assume dual currency equilibrium with asymmetric acceptability ($\Delta(\alpha_2) = 0$). Then, directly from (33), $\alpha_2 \in (0, 1)$ implies $\gamma_d > \gamma_f$. ■

In a dual currency economy with asymmetric acceptability, the money with a lower rate of growth provides a better protection against the inflation tax, but is less accepted for payment in the DM. Conversely, the money with a higher rate of growth has a higher acceptability value, but provides less protection against the inflation. In this economy, the net cost (in terms of higher inflation tax) of holding the money with a higher rate of growth must be exactly compensated by its net benefit (in terms of higher liquidity). In a dual currency economy with symmetric acceptability, the two monies must grow at the same rate, so they provide the same degree of protection from the inflation tax.

4 Extension: illegal currency

We now modify our setup in two directions. First, we relax the information problem introduced in the previous section. Specifically, we assume that any agent can perfectly recognize counterfeit from genuine foreign currency. This implies that nobody engages in counterfeiting activity anymore. Second, we assume that the use of foreign currency as a medium of exchange is prohibited. That is, type-2 transactions are illegal. Anyone can carry the foreign money, but it is illegal to use it for buying goods in the DM. To discourage agents engaging in the illegal activity, we introduce enforcement and punishment as follows.

At the beginning of each period, before the preference/consumption shocks realize, each agent

receives an idiosyncratic shock such that he is a Cop (\mathcal{C}), with probability λ , and he is not a Cop ($\bar{\mathcal{C}}$) with a probability $1 - \lambda$. Type- \mathcal{C} agents act like type- $\bar{\mathcal{C}}$ ones when in a type-1 meeting –i.e. they produce or consume special goods in exchange for domestic money– but when they find themselves in a type-2 meeting they don't trade, rather they confiscate the partner's portfolio of monies. Note that carrying the foreign currency is not prohibited per se, only the act of paying or getting paid in foreign currency is prosecuted by type- \mathcal{C} agents.

The value function for an agent entering the DM with a portfolio \mathbf{m} of monies is therefore

$$V(\mathbf{m}) = \lambda V^{\mathcal{C}}(\mathbf{m}) + (1 - \lambda) V^{\bar{\mathcal{C}}}(\mathbf{m}). \quad (34)$$

The value function for a type- \mathcal{C} agent is

$$\begin{aligned} V^{\mathcal{C}}(\mathbf{m}) = & \sigma\alpha_1[u(q_1) + W(\mathbf{m} - \mathbf{z}_{1,b})] + \sigma\alpha_2(1 - \lambda)W(2\mathbf{m}) + \sigma\alpha_2\lambda W(\mathbf{m}) \\ & + \sigma\alpha_1[-c(q_1) + W(\mathbf{m} + \mathbf{z}_{1,s})] + \sigma\alpha_2(1 - \lambda)W(2\mathbf{m}) + \sigma\alpha_2\lambda W(\mathbf{m}) \\ & + (1 - 2\sigma)W(\mathbf{m}) \end{aligned} \quad (35)$$

A type- \mathcal{C} agent can be a buyer with probability σ (first row), in which case he acquires and consumes special goods using domestic money if he is in a type-1 meeting (first term), or he confiscates the partner's portfolio if he is in a type-2 meeting (first and second term); note that if the type- \mathcal{C} 's partner in a type-2 meeting is of type- \mathcal{C} , they both confiscate each other (third term). The second row is the expected value of a type- \mathcal{C} agent being a seller in the DM, in which case he produces and sells special goods in exchange for domestic money if he is in a type-1 meeting (first term), or he confiscates the partner's portfolio if he is in a type-2 meeting (second and third term). Again, if the type- \mathcal{C} 's partner in a type-2 meeting is of type- \mathcal{C} , they confiscate each other (third term). The third row is the expected value of a type- \mathcal{C} agent being a nontrader in the DM.

The value function a type- $\bar{\mathcal{C}}$ agent is

$$\begin{aligned} V^{\bar{\mathcal{C}}}(\mathbf{m}) = & \sigma\alpha_1[u(q_1) + W(\mathbf{m} - \mathbf{z}_{1,b})] + \sigma\alpha_2(1 - \lambda)[u(q_2) + W(\mathbf{m} - \mathbf{z}_{2,b})] + \sigma\alpha_2\lambda W(\mathbf{0}) \\ & + \sigma\alpha_1[-c(q_1) + W(\mathbf{m} + \mathbf{z}_{1,s})] + \sigma\alpha_2(1 - \lambda)[-c(q_2) + W(\mathbf{m} + \mathbf{z}_{2,s})] + \sigma\alpha_2\lambda W(\mathbf{0}) \\ & + (1 - 2\sigma)W(\mathbf{m}) \end{aligned} \quad (36)$$

The first row means that the type- $\bar{\mathcal{C}}$ agent can be a buyer, in which case he consumes special goods in exchange for domestic money if he finds himself in a type-1 meeting (first term), or he

can consume special goods in exchange for the foreign and domestic money if he find himself in a type-2 meet with a type- \bar{C} agent (second term), or he gets his portfolio confiscated if he is in a type-2 meeting with a type- C agent (third term). The second row is the expected value of a type- \bar{C} agents who is a seller in the DM, which means that he produces special goods for domestic money if he is in a type-1 meeting (first term), or he produces special goods in exchange for the domestic and foreign money if he is in a type-2 meeting with a type- \bar{C} agent (second term), or he gets all his money confiscated if he is in a type-2 meeting with a type- C agent (third term). The third row is the expected value of a type- \bar{C} agent who is a nontrader in the decentralized market.

Replacing $V^C(\mathbf{m})$ and $V^{\bar{C}}(\mathbf{m})$ using (35), respectively (36), and rearranging terms, we can rewrite (34) as follows

$$\begin{aligned} V(\mathbf{m}) &= \sigma\alpha_1 [u(q_2) - \phi\mathbf{z}_{2,b}] + \sigma\alpha_1 [-c(q_2) - \phi\mathbf{z}_{2,s}] \\ &\quad + \sigma\alpha_2 (1 - \lambda)^2 [u(q_2) - \phi\mathbf{z}_{2,b}] + \sigma\alpha_2 (1 - \lambda)^2 [-c(q_2) - \phi\mathbf{z}_{2,s}] \\ &\quad + W(\mathbf{m}). \end{aligned} \tag{37}$$

where we have used linearity of $W(\mathbf{m})$. Next, take the differential of (37) with respect to m_d and m_f and get

$$V_d(\mathbf{m}) = \phi_d \sigma \alpha_1 \left[\frac{u'(q_1)}{g'(q_1)} - 1 \right] + \phi_d \sigma \alpha_2 (1 - \lambda)^2 \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right] + W_d(\mathbf{m}) \tag{38}$$

and,

$$V_f(\mathbf{m}) = \phi_f \sigma \alpha_2 (1 - \lambda)^2 \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right] + W_f(\mathbf{m}) \tag{39}$$

The right side of (38) is the marginal benefit of carrying one unit of domestic currency in the DM. The first term means that an additional unit of the domestic currency can be used to acquire goods in a type-1 meeting. The second term means that an additional unit of domestic money can be used for goods purchases in a type-2 meeting (second term), which is the case if and only if partners are both of type- \bar{C} ; note that for confiscation to occur at least one agent in a type-2 meeting must be of type- C . Equation (39) refers to the marginal value of the foreign currency. The first term on the right side means that an additional unit of the foreign currency can be used for goods purchases in a type-2 meeting given both agents are of type- \bar{C} . Using (9) and the second condition in (8) lagged

one period, the last two equations can be rewritten as

$$\frac{\gamma_d - \beta}{\beta} = \sigma \left\{ \alpha_1 \left[\frac{u'(q_1)}{g'(q_1)} - 1 \right] + (1 - \alpha_1) (1 - \lambda)^2 \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right] \right\} \quad (40)$$

and

$$\frac{\gamma_f - \beta}{\beta} = \sigma (1 - \alpha_1) (1 - \lambda)^2 \left[\frac{u'(q_2)}{g'(q_2)} - 1 \right] \quad (41)$$

Note that a type- \bar{C} seller faces the probability λ of getting caught in the act of getting paid in a type-2 meeting, in which case he gets his portfolio of monies $\phi \mathbf{m}$ confiscated. Hence, his net expected benefit of being in a type-2 meeting, as opposed to a type-1 meeting, is now

$$\Delta \equiv (1 - \lambda) [g(q_2) - c(q_2)] - \lambda \phi \mathbf{m} - g(q_1) + c(q_1)$$

where, the first term in the right hand side is the probability of being in a type-2 meeting with a type- \bar{C} buyer times the surplus from trade, the second term is the probability of being caught times the amount of the fine, and the third term is the trade surplus in a type-1 meeting. Using the fact that (24) is always binding and by virtue of $\phi \mathbf{z}_2 = g(q_2)$, the above expression can be rewritten as follows

$$\Delta \equiv (1 - \lambda) [g(q_2) - c(q_2)] - \lambda g(q_2) - g(q_1) + c(q_1). \quad (42)$$

Note that we can have three possible equilibria, depending on the sign of Δ . They are described below as follows.

Equilibrium with type-1 and type-2 meetings.

A seller is indifferent between accepting only the domestic currency and accepting both the domestic and the foreign currency when his net benefit of being in a type-1 meeting, as opposed to a type-2 meeting, is zero, i.e. $\Delta = 0$. In any equilibrium where $\Delta = 0$ we have that $\alpha_1 \in [0, 1]$. He will be in a type-1 meeting with a probability α_1 , and he will be in a type-2 meeting with the complement probability $1 - \alpha_1$.

Definition 4 *An equilibrium with both type-1 and type-2 meetings is a path (q_1, q_2, α_1) satisfying (40), (41), and (42) given parameters $(\beta, \sigma, \lambda, \gamma_d, \gamma_f)$, with $\Delta = 0$ and $\alpha_1 \in [0, 1]$.*

Equilibrium with type-1 meetings only.

An equilibrium with type-1 meetings only occurs when the seller has no incentive to be in a

type-2 meeting, as opposed to a type-1 meeting, that is when $\Delta < 0$. In a symmetric equilibrium, $\Delta < 0$ implies $\alpha_1 = 1$. In this type of equilibrium agents don't want to hold the foreign currency since nobody will accept it for payment. Hence, the domestic money is the only medium of exchange in the DM and the expression (40) becomes

$$\frac{\gamma_d - \beta}{\beta} = \sigma \left[\frac{u'(q)}{g'(q)} - 1 \right]. \quad (43)$$

Definition 5 *An equilibrium with type-1 meetings only is a q satisfying (43) given parameters value $(\beta, \sigma, \gamma_d)$, with $\Delta < 0$ and $\alpha_1 = 1$.*

Since $u'(q)/g'(q)$ is continuous and decreasing in q then there exists a q that satisfies (43) and the solution is unique.

Equilibrium with type-2 meetings only.

If $\Delta > 0$, a seller will find more profitable to accept both the domestic and foreign currency for payment, instead of accepting the domestic currency only. In a symmetric equilibrium, $\Delta > 0$ implies $\alpha_1 = 0$. That is, everybody holds the foreign currency because nobody accepts only the domestic currency for payment when $\Delta > 0$. If $\Delta > 0$, both currency are symmetrically accepted, that is they have the same liquidity value. Replacing α with 1, expressions (40) and (41) can be rewritten as

$$\frac{\gamma_d - \beta}{\beta} = \sigma (1 - \alpha_1) (1 - \lambda)^2 \left[\frac{u'(q)}{g'(q)} - 1 \right], \text{ and} \quad (44)$$

$$\frac{\gamma_f - \beta}{\beta} = \sigma (1 - \alpha_1) (1 - \lambda)^2 \left[\frac{u'(q)}{g'(q)} - 1 \right] \quad (45)$$

respectively.

Definition 6 *An equilibrium with type-2 meetings only is a q satisfying (44) and (45) given parameters value $(\beta, \sigma, \lambda, \gamma_d, \gamma_f)$, with $\Delta > 0$ and $\alpha_1 = 0$.*

Clearly, an equilibrium with type-2 meetings only exists if $\gamma_d = \gamma_f$. The intuition is that if both currencies have the same liquidity value they also need to provide the same protection against the inflation tax in order for agents to hold both of them.

5 Conclusions

[To be added]

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